# Ride the Tide of Traffic Conditions: Opportunistic Driving Improves Energy Efficiency of Timely Truck Transportation

Wenjie Xu, Qingyu Liu, *Member, IEEE,* Minghua Chen, *Fellow, IEEE,* and Haibo Zeng, *Member, IEEE*

***Abstract*—We study the problem of minimizing fuel consumption of a heavy-duty truck traveling across the national highway network subject to a hard deadline. We focus on a real-world setting that traversing a road segment is subject to variable speed ranges due to dynamic traffic conditions. The consideration of dynamic traffic conditions not only differentiates our work from existing ones but also allows us to leverage *opportunistic driving* to improve fuel efficiency. The idea is for the truck to strategically wait (e.g., at highway rest areas) for benign traffic conditions, so as to traverse subsequent road segments at favorable speeds for saving fuel. We observe that traffic conditions and thus speed ranges are mostly stationary within certain duration of the day, and we term them as *phases*. We formulate the fuel consumption minimization problem under phased speed ranges, considering path planning, speed planning, and opportunistic driving. We prove that the problem is NP-hard, and develop a dual-subgradient algorithm for large-/national- scale instances. We characterize conditions under which the algorithm generates an optimal solution. We carry out simulations based on real-world traces over the US highway system. The results show that our scheme saves up to $20\%$ fuel than a shortest-path based alternative, of which opportunistic driving contributes $13\%$. Meanwhile, opportunistic driving also reduces driving time by $6\%$ as compared to only optimizing path planning and speed planning. As such, it offers a desirable design option to *simultaneously* reduce fuel consumption and hours of driving. Last but not least, our results highlight a perhaps surprising observation that dynamic traffic conditions can be exploited to achieve fuel savings even *larger* than those under stationary traffic conditions.**

***Index Terms*—Energy-efficient transportation, timely transportation, opportunistic driving, dynamic traffic conditions, variable speed ranges**

## I. INTRODUCTION

The United States (US) trucking industry hauls $70\%$ of all freight tonnage and earns \$700.1 billion in gross freight revenues in 2017 [2]. If measured against the countries' gross domestic products (GDPs), the impressive revenue would rank 19 worldwide. Meanwhile, heavy-duty trucks consume $18\%$ of energy in the whole transportation sector [3], although they only account for $4\%$ of the total vehicle population. Furthermore, a large fraction (26˜34%) of the truck operation cost is contributed by fuel consumption [3]. In addition, it is forecasted that global freight activity will increase by a factor of 2.4 by 2050 [4]. These observations make it critical to reduce fuel consumption for heavy-duty truck operation.

In this paper, we study an essential problem in the area of long-haul heavy-duty truck operation, i.e., minimizing fuel consumption of a heavy-duty truck traveling across national highway network subject to a hard deadline constraint, under a real-world setting that traversing a road segment is subject to variable speed ranges due to dynamic traffic conditions.

*Transportation deadline*: it is common to have time guarantee for freight delivery in truck operation; see examples and discussions in [5], [6]. As a more recent example, mobile applications like Uber Freight provide many freight transportation tasks for truck operators, often associated with pickup/delivery time requirements.

*Traffic condition*: In practice, the speed ranges that a truck can travel on a highway depend on the dynamic traffic condition, especially on the highways near urban areas. For example, during the rush hour, such as in the morning, one may only be able to drive at a speed much lower than the regulatory speed limit due to harsh traffic conditions. In contrast, during the off-peak hours, the traffic conditions are more favorable. Consequently, one can drive around the regulatory speed limit. The dynamic traffic conditions lead to *variable speed ranges* (VSR) of driving.

*Path planning* and *speed planning* are two well-recognized approaches to reducing fuel consumption. When driving along different paths, differences in distances and road conditions such as grade can lead to substantially different fuel consumption (e.g., as much as $21\%$ according to [7]). Meanwhile, it is also critical to drive at an appropriate speed to save fuel, considering that there is usually a most fuel-efficient speed for each vehicle. It is about $55$ mph (mile per hour) for many trucks [8], and the fuel economy will deteriorate when driving at a lower or higher speed. As reported by [9], [10], every one mph increase in speed (above the most fuel-efficient speed) causes about 0.14 mpg (mile per gallon) decrease in fuel economy. Meanwhile, it is also reported in [11] that, when traffic congestion causes the average speed to be below 45mph, there is a negative impact on vehicle's carbon dioxide emissions (and thus fuel consumption) in general. We highlight that road grade also plays an essential role in speed planning. This is because, with different grades, the fuel-rate-speed

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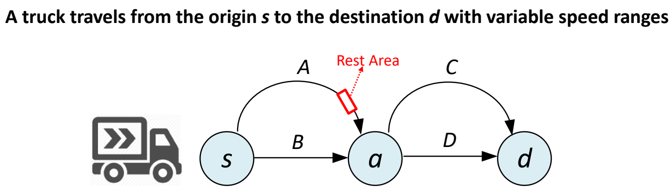



| Time Interval | [0,1) | [1,2) | [2,3) |
|---|---|---|---|
| Speed Range of *A*, *B*, *C* | [30,50] | [30,40] | [30,50] |
| Speed Range of *D* | [30,50] | [30,35] | [30,50] |

| | Solution | Fuel Cost | Driv. Time |
|---|---|---|---|
| w/o O.D. | $s \xrightarrow{B,50} a \xrightarrow{C,40} d$ | 3.0 | 2.25 |
| w. O.D. | $s \xrightarrow{A,50}$ wait $\rightarrow a \xrightarrow{D,50} d$ | 2.0 | 2.0 |

Fig. 1: A truck departs from $s$ at time 0 and needs to arrive at $d$ by time 3. There is a rest area at the end of road segment $A$. Each road segment has a length of 50 units and a corresponding fuel consumption rate function $f(r) = 0.01 \times (r-50)^2+1$, where $r$ is the traveling speed. The first table shows the dynamic speed ranges for each road segment. As shown in the second table, opportunistic driving (O.D.) reduces fuel consumption and driving time simultaneously in this example.

functions are different. As pointed out in [12], the diversity of grades brings the potential for reducing fuel consumption without increasing trip time.

In addition to path planning and speed planning, the consideration of dynamic traffic conditions allows us to leverage *opportunistic driving* to further improve fuel efficiency. The idea is for the truck to strategically wait for benign traffic conditions, to traverse subsequent road segments at favorable speeds to save fuel. [1] Specifically, in the US, there are rest areas with parking facilities located next to highways [14]. As many as $2.3\%$ of the highway road segments are associated with a truck rest area; see Sec. VIII for a rough analysis. Drivers can refuel, rest, or eat without exiting onto secondary roads [15] in these rest areas. It can be more fuel-efficient for a truck driver to wait at certain rest areas for an appropriate amount of time, such that he/she can avoid the harsh traffic conditions and traverse subsequent road segments at favorable speeds to save fuel. We note that in practice, truck drivers usually only need to deliver the loads to the destination before a given deadline; see, e.g., the freight transportation requests on Uber Freight [16] and uShip [17]. Early arrival does not bring additional economic benefit. In such cases, drivers may prefer to trade a longer trip time (not necessarily longer driving time) for fuel reduction, as much as 20% as compared to conceivable shortest-path with fastest speed alternative; see Sec. VIII for more details.

We give an illustrative example and the optimal solution without (resp. with) opportunistic driving in Fig. 1. Without opportunistic driving, the optimal solution is a path of $\langle B, C\rangle$ with speed $\{r_B = 50, r_C = 40\}$. This solution has a total fuel consumption of 3. In contrast, with opportunistic driving, the optimal solution is a path of $(A, D)$ with speed $\{r_A = r_D = 50\}$, and rest for one unit of time after traversing $A$ before entering $D$. It results in the total fuel consumption of 2. Note that opportunistic driving also reduces driving time from 2.25 to 2.0 in this example.

[1] At a high level, the idea is similar to the method of opportunistic scheduling widely used in commercial cellular wireless systems [13], in the sense that both aim to exploit variation in the environment to improve system performance. In wireless communication, opportunistic scheduling exploits the communication channel gain's variation to increase the throughput. In our scenario of truck transportation, we propose to leverage the dynamic traffic conditions to drive opportunistically to reduce fuel consumption.

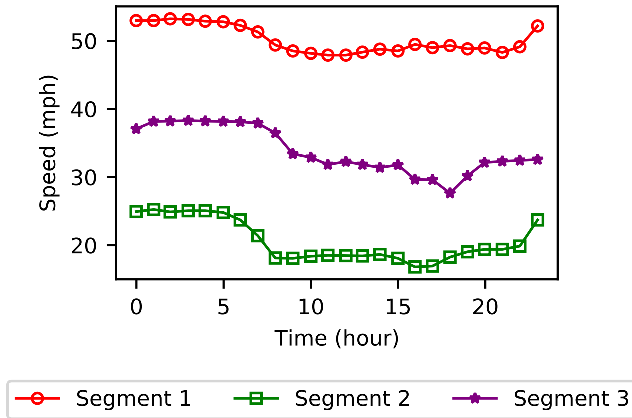



Fig. 2: The average traffic speeds in a single day on three randomly selected road segments in the eastern US highway network.

We also justify the temporal-spatial diversity of traffic conditions using real-world data as follows. Similar to [18]–[21], we model the fuel consumption as a function of driving speed. We randomly select a road segment in the eastern US, and we collect the road speed data using the traffic application programming interface (API) provided by HERE, a location data service platform [22] (similar to those in Fig. 2). Fig. 3 shows that at 9 PM, the driving speed range corresponds to a less fuel-efficient part of the fuel-speed function. However, one hour later, at 10 PM, the traffic condition improves, and the speed range changes, allowing the truck to travel at a higher and more fuel-efficient speed to save fuel. Fig. 2 gives the average traffic speeds across one day of three randomly selected road segments in the eastern US highway (averaged over ten days using the data from the HERE map [22]). We observe that traffic speed shows certain "phase" properties. For example, all the traffic speeds are high during off-peak hours, e.g., from 3 AM to 6 AM, and are relatively low during peak hours, e.g., from 9 AM to 12 PM. As shown in Fig. 2, the traffic condition also shows spatial diversity, which can be readily exploited. By opportunistically scheduling the truck to traverse busy road segments (like road segments near a city or in the urban area) during the off-rush hour, strategically rest at rest areas, or drive on free roads (like the ones in the village) during rush hour, the truck can travel at more energy-efficient speeds to reduce fuel consumption.

Overall we observe that under dynamic traffic conditions, it is critical to jointly consider path planning, speed planning, and opportunistic driving.

In this paper, we study the problem of minimizing fuel consumption of a truck travelling from an origin to a des-

| Existing Work | Design Space | | | Constraint | |
|---|---|---|---|---|---|
| | Path Planning | Speed Planning | Opportunistic Driving | Deadline | Speed Range |
| [23]–[25] | ✓ | ✗ | ✗ | ✓ | N/A |
| [18], [19] | ✓ | ✓ | ✗ | ✓ | Static |
| [20], [21] | ✓ | ✓ | ✗ | ✓ | Static |
| [26] | ✗ | ✓ | ✗ | ✗ | Static |
| [27] | ✓ | ✗ | ✗ | ✗ | N/A |
| This paper | ✓ | ✓ | ✓ | ✓ | Variable |

TABLE I: Comparison of existing studies for optimizing energy-efficient truck transportation.

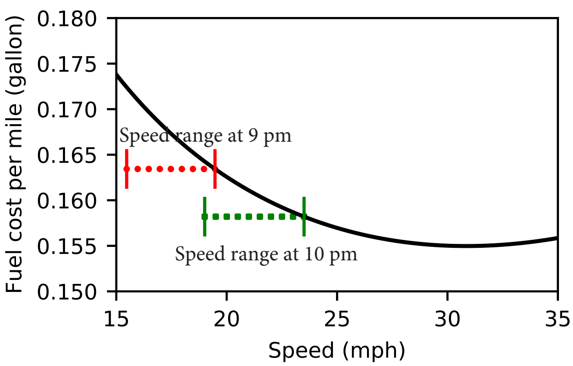



Fig. 3: The normalized fuel-speed function of a loaded truck for a road segment. The most fuel-economic speed is 32 mph.

tination across a highway network under a hard deadline constraint. We optimize path planning, speed planning, and opportunistic driving simultaneously under Variable Speed Range constraints that result from dynamic traffic conditions. We remark that our solution allows fuel saving under the condition that the truck meets the deadline, which is an input to our algorithm. Hence, the driver or truck can set the deadline according to their own considerations. We make the following contributions.

▷ We introduce the new design space *opportunistic driving*. Our study shows that with opportunistic driving, it is possible to simultaneously reduce the driving time and the energy consumption, by strategically driving during favorable traffic conditions. This results in highly desirable operating profiles previously not possible, and debunks the commonly believed myth that energy savings are only possible by increasing the driving time [18]–[21].

▷ We observe that the traffic conditions and hence the speed ranges are mostly stationary within certain periods of the day [28], [29]. We term them as phases. Each phase is a time interval with fixed speed ranges for each road segment. The concept of phase is consistent with the classical traffic theory [30]. We also justify it with real-world data. The concept of phase captures the dynamic and periodic characteristics of traffic conditions. Leveraging the phase concept, we develop a phase-expanded-graph based formulation. Within this formulation, speed range is fixed in the same phase but differs across phases, making our problem more tractable while still capturing the dynamics of traffic condition.

▷ We prove that our problem is NP-hard. We then exploit the structure of the dual of our phase-based formulation to design an efficient algorithm. Our algorithm can obtain high-quality solutions efficiently for large-/national- scale instances. We further derive a sufficient condition under which our algorithm generates an optimal solution.

▷ We conduct extensive simulations using real-world traces over the US highway network. The results show that our solutions save up to 20% fuel compared to the shortest path based alternative, of which 13% is contributed by opportunistic driving. Meanwhile, opportunistic driving also reduces the hours of driving by 6% compared to only optimizing path and speed. As such, opportunistic driving offers a favorable design option for truck drivers to *simultaneously* reduce fuel consumption and hours of driving. Simulation results also show that our solution is robust to uncertain traffic conditions.

## II. Related Work

Energy-efficient trucking has long been an active research area with efforts on various design options, including path planning, speed planning, platooning, and autonomous driving, etc. To the best of our knowledge, this is the first study to consider the new design space of opportunistic driving by *exploiting* dynamic traffic conditions in addition to the two existing design spaces of path planning and speed planning. Tab. I compares our work with most related studies.

**Restricted Shortest Path (RSP).** This problem is about finding a path from an origin to a destination, to minimize total cost subject to a deadline constraint. RSP is NP-hard [23], and researchers have designed both approximation algorithms with performance guarantee [23] and heuristic schemes [25] for solving RSP problems. Meanwhile, the RSP setting does not consider speed planning as the driving speed is fixed on each road segment and cannot be changed.

**PAth selection and Speed Optimization (PASO).** The PASO problem [18], [19] is to find a path from an origin to a destination and the corresponding travel speeds along the path, so as to minimize the fuel cost subject to a hard deadline constraint. It generalizes the RSP problem by considering speed optimization and speed-dependent cost functions. Meanwhile, the PASO problem setting does not consider dynamic speed ranges, which may well exist in practice, or opportunistic driving.

**Timely tRansportation for Energy-efficient trucKing (TREK).** TREK [20] is an extension of PASO [18], [19]. Given many origin-destination pairs, it requires to find an origin-to-destination path for each pair, to minimize total fuel cost subject to individual time window constraints of individual pairs. Under a particular setting where there is only one origin-destination pair, TREK is equivalent to PASO.

There are also related works in the general area of energy-efficient vehicle transportation.

**Time-Dependent Path and Speed Planning.** Besides the research focusing on energy-efficient truck transportation, there have been lots of efforts devoted to the area of general Vehicle Routing Problem (VRP). One important extension of VRP problem is time-dependent VRP problem [31]–[33] where the vehicle speed is a function of time. Among the Time Dependent Vehicle Routing Problem (TDVRP) literature, strategical waiting is shown to reduce emissions in a very simple artistic network [33]. However, the approach in [33] either discretizes speed or fixes route, so it does not optimize route, speed, and waiting time jointly.

**Fuel minimization in truck operations.** Operating trucks in a more fuel-economic way has been a major approach to reducing truck energy consumption. Many design options have been explored, e.g., platooning two or more trucks [34]–[36] and reducing idling energy consumption [37]. In addition, many works also separately optimize the route planning [38], [39] and speed planning [26], [40] of trucks to reduce fuel consumption.

**Energy-aware routing.** Energy-aware routing problem has been receiving more and more attention. The authors in [41] introduce the Green Vehicle Routing Problem (GVRP), which aims to overcome the challenges of limited driving range and limited refueling infrastructure. Lin *et al.* [42] give a thorough survey of the area of Green Vehicle Routing Problem. One subarea that is more closely related to our work is the Energy Minimizing Vehicle Routing Problem (EMVRP) [43], where the authors propose to minimize the energy consumption of vehicles in VRP problem. Other VRP-related research that aims to minimize fuel consumption include [44]–[46].

**Just-in-time routing.** Just-in-time routing problem aims to deliver goods around the delivery time by adjusting driving speed and traveling path [47] [48] or by optimizing the pickup and delivery schedules [49]. Differences between our study and existing just-in-time ones are the following: (i) In our study, the deadline constraint is a hard constraint that is not allowed to be violated. In contrast, just-in-time studies usually allow violating the deadline with a tardiness cost. (ii) Our study explores the entire design space of opportunistic driving for saving fuel, where trucks can strategically wait in rest areas for benign traffic conditions. This design space has not been investigated in existing just-in-time studies, and suggests a new important design option for solving other vehicle delivery problems with dynamic traffic conditions. For example, one can consider opportunistic driving when adjusting vehicle traveling profiles based on real-time traffic information towards just-in-time delivery with lower fuel consumption.

**Fuel cost function modelling** Fuel cost of vehicles is affected by many factors. The main factors include travel time, distance, weather, vehicle design, roadway condition, traffic condition, and driving behavior, etc [50]. Therefore, it's difficult to precisely model and predict the fuel consumption of vehicles. One approach to model fuel consumption is deriving the fuel consumption from the first principle [51]–[54]. For example, Cachón, *et al.*, in [51] used the carbon balance method to predict the fuel consumption of a natural gas vehicle. However, such a first-principle based method requires an understanding of physics and chemical knowledge related to the vehicle. Another approach is the black box method [50], where the vehicle is viewed as a black box and a mathematical model of fuel consumption is built from the input-output data [26], [55]–[61]. For example, Rakha and Ahn [61] develop a mathematical model where the instantaneous fuel consumption rate is modelled as a function of instantaneous speed and instantaneous acceleration.

Moreover, Hellstrom *et al.* [26] develop an assistance system that uses the predicted grade information to optimize driving speed to save fuel. However, they assume a fixed path and hence do not optimize path planning. Boriboonsomsin *et al.* [27] present an eco-routing navigation system that can determine the most fuel-economic path. However, they assume fixed road driving speeds and hence do not optimize speed planning. Moreover, existing studies [26], [27] assume static road speed ranges and do not consider opportunistic driving.

In addition to path planning and speed planning, there are studies exploring other potentials, e.g., autonomous driving [62], [63], and vehicle platooning [64]–[66], for saving fuel. Here we remark that the design options explored in [62]–[66] for fuel consumption reduction differ from our options of path planning, speed planning, and opportunistic driving. Note that our solutions provide an energy-efficient path and speed profile for individual long-haul timely truck transportation and, thus, can serve as a critical building block for the whole energy-efficient truck transportation system.

As compared to [1], this paper contains substantial new results. First, we include the convergence rate analysis of the proposed dual-subgradient algorithm. Second, we propose methods of constructing traffic phases from real-world speed data. Third, we present simulation results under more practical settings with real-world rest area data. Last but not least, this paper highlights the practicability and strong performance of our proposed opportunistic driving in both saving fuel and reducing driving time.

## III. System Model and Problem Formulation

### A. System Model

We model a national highway network as a directed graph $G \triangleq (V, E)$, where $E = E_s \bigcup E_r$. An edge $e \in E_s$ represents an actual road segment. An edge $e \in E_r$ is a virtual edge modelling a rest area where drivers can park their trucks and rest. Thus waiting at the rest area is modeled as traveling along a virtual edge. Node $v \in V$ represents a connecting point. A node can connect two actual road segments or connect a rest area with an actual road segment. For an edge $e = (u, v) \in E$, we use $\mathsf{head}(e)$ to denote its head, i.e., the node $v$, and use $\mathsf{tail}(e)$ to denote its tail, i.e., the node $u$.

Note that a road segment in practice may correspond to multiple segments in the modeled directed graph. For example, for a 3-mile road with a rest area one mile away from the entry point of the road, we represent it by three edges in order $\{e_1, e_2, e_3\}$: $e_1 \in E_s$ corresponds to the first 1-mile segment of this road, $e_2 \in E_r$ corresponds to the rest area, and $e_3 \in E_s$ corresponds to the remaining 2-mile segment.

For each $e \in E_s$, let $D_e$ denote its length, and let $R_e^{lb}(t)$ (resp. $R_e^{ub}(t)$) denote its minimum (resp. maximum) speed limit that depends on the time $t$ when the truck enters $e$. We remark that VSR, i.e., the time-varying speed ranges $[R_e^{lb}(t), R_e^{ub}(t)]$, differentiates our work from existing studies, e.g., [18]–[20]. Further, to model opportunistic driving, we explicitly consider a rest edge set $E_r$ where the truck can strategically wait for benign traffic conditions. Existing studies [18]–[20] only consider stationary traffic conditions where $R_e^{lb}(t) = R_e^{lb}$, $R_e^{ub}(t) = R_e^{ub}$ for each $e \in E = E_s$, i.e., speed ranges are fixed on different edges and do not change over time. While this is the case for highways in remote areas that have little population and traffic, constant speed range does not properly model the dynamic traffic conditions that are common in urban or metropolitan areas; see an example in Fig. 2. In contrast, we incorporate both space variation and time variation of speed range into our model and capture the dynamics of traffic conditions.

In this paper, we consider a fuel-efficient truck transportation problem under Variable Speed Range with a hard deadline constraint. Truck fuel consumption is determined by many factors, e.g., road grade and driving speed. Similar to the studies in [18]–[20], we assume for each edge $e \in E$, all the environment-specific factors such as road grade are fixed. Thus given the truck's weight and a specific edge, we can model the truck fuel consumption rate as a convex function of driving speed in a reasonable range as justified in [18], [19]. Because of the convexity, it is sufficient to maintain a constant speed when driving along a road segment (see [18, Lem. 1]), without loss of optimality. Meanwhile, similar to [18]–[20] and as justified in Sec. VB of [21], we neglect the time and fuel consumption of the acceleration/deceleration stage when a truck changes its speed across adjacent edges, as this stage is usually only several hundred meters long [67], and the corresponding time and fuel consumption are negligible compared to those of traveling along one road segment which is usually several miles long. To further cross verify our assumption, we use Future Automotive Systems Technology Simulator (FASTSim) [68] to simulate a class 8 truck's fuel consumption under two different scenarios. In the first scenario, the truck travels through a road segment with length $d$ at the constant speed of $v$. In the second scenario, the truck first accelerates from speed 0 mph to speed $v$, then keeps the constant speed $v$ and finally decelerates to 0 mph. We set $d = 6$ km, $v = 36$ km/h. And we set the driving time used for acceleration and deceleration both to be 60s. The simulation result shows that the fuel cost impact of acceleration/deceleration is less than $1\%$, which is much less than the fuel saving achieved by our solution (that will be seen in our simulation) and therefore we ignore it for modelling simplicity.

Overall, we use $f_e(r_e)$ to denote the truck's fuel consumption rate function to traverse an edge $e \in E$, at a constant speed of $r_e$. We assume that (i) $f_e(r_e) = 0, \forall e \in E_r$, (since there is no fuel consumption at rest area) and (ii) $f_e(r_e)$ is strictly convex in $r_e$ over a reasonable speed range for any $e \in E_s$ (see the discussions in [18]–[20]). With the fuel-rate function $f_e(r_e)$, we can define the fuel consumption function $c_e(t_e)$:

$$c_e(t_e) = \begin{cases} t_e \cdot f_e\left(D_e/t_e\right), & \text{if } e \in E_s, \\ 0, & \text{otherwise (i.e., } e \in E_r\text{)}, \end{cases} \tag{1}$$

which is the fuel consumption for the truck to traverse an edge $e \in E$, with a travel time of $t_e$. We note that the charge at a rest area $e \in E_r$ can be taken into account by setting the $c_e(t_e)$ to be a constant or a linear function. Our model and algorithm (to be introduced as Alg. 1 in Sec. VI) in this paper still applies, although the solution obtained for specific instances may vary.

### B. Problem Formulation

We consider the scenario where a truck travels from an origin $s \in V$ to a destination $d \in V$ over a national highway network $G$. Our objective is to *minimize the total fuel consumption* subject to *Variable Speed Range constraints* and *a hard deadline constraint*. VSR constraints require that the truck driving speed at time $t$ on each edge $e \in E_s$ must be no smaller than $R_e^{lb}(t)$ and no larger than $R_e^{ub}(t)$. The deadline constraint requires that the total travel time from $s$ to $d$, including the truck driving time on edges $e \in E_s$ and the truck waiting time on edges $e \in E_r$, must not exceed a given deadline $T$.

The design space includes path planning, speed planning, and opportunistic driving. We introduce the following decision variables: variable $p$ defines a simple path from $s$ to $d$ over the graph $G$, and variable $t_e$ defines the time for the truck to traverse an edge $e \in E$ [2]. By vectoring variables as $\vec{t} \triangleq \{t_e : \forall e \in E\}$, our problem of optimizing Path planning, Speed Planning, and opportunistic driving to save fuel under VSR (PSPV) can be formulated as follows:

$$\mathsf{PSPV}: \quad \min_{p \in \mathcal{P}, \vec{t} \in \mathcal{T}_p} \quad \sum_{e \in p} c_e(t_e), \tag{2a}$$

$$\text{s.t.} \quad \sum_{e \in p} t_e \leq T, \tag{2b}$$

where $\mathcal{P}$ is the set of simple paths from $s$ to $d$ in $G$, and $\mathcal{T}_p$ defines the set of possible travel time profile of all edges on the path $p$, i.e.,

$$\mathcal{T}_p \triangleq \Big\{ \vec{t} : R_e^{lb}\left(\mathcal{S}_e(p, \vec{t})\right) \leq D_e/t_e \leq R_e^{ub}\left(\mathcal{S}_e(p, \vec{t})\right), \\ \forall e \in E_s : e \in p \; ; \; 0 \leq t_e \leq T, \forall e \in E_r : e \in p \Big\}, \tag{3}$$

where

$$\mathcal{S}_e(p, \vec{t}) = \begin{cases} 0, & \mathsf{pre}(e,p) \text{ is empty;} \\ \sum_{e' \in \mathsf{pre}(e,p)} t_{e'}, & \text{otherwise.} \end{cases} \tag{4}$$

[2]If $e \in E_s$, $t_e$ is driving time; otherwise if $e \in E_r$, $t_e$ is resting/waiting time.

Here $\mathsf{pre}(e,p)$ is the set of edges that are on the path $p$ and are precedent to the edge $e$, i.e., if the ordered edges of path $p$ is $\langle e_1, e_2, ..., e_{|p|}\rangle$, then we have:

$$\mathsf{pre}(e_k,p) \;=\; \{e_j : \forall j = 1,2,...,k-1\}, \quad \forall k = 1,2,...,|p|. \tag{5}$$

With $\mathsf{pre}(e,p)$, $\mathcal{S}_e(p,\vec{t})$ is the starting time for the truck to traverse the edge $e$ following the path $p$ and the edge travel time defined by $\vec{t}$.

In the formulation in (2), the objective in (2a) minimizes the fuel consumption and constraint in (2b) restricts the total travel time, including the total driving time and the total waiting time, by the deadline $T$. Note that $\vec{t} \in \mathcal{T}_p$ are Variable Speed Range constraints already defined before.

We remark again that PSPV sets objective of saving fuel without missing the deadline, where the deadline is input by the user (e.g., trucking company, truck driver) according to their own considerations. As discussed in Sec. I, in many real-world scenarios, truck drivers only need to deliver the freight to the destination before a given deadline and there is little further economic gain for drivers to arrive early. In these scenarios, drivers may choose to trade a longer trip time for fuel reduction.

Our PSPV is NP-hard, since its special case PASO under stationary traffic conditions is already NP-hard [18]. Thus it is impossible to obtain an optimal solution for PSPV in polynomial time, unless P = NP.

**Proposition 1.** *Problem PSPV is NP-hard.*

## IV. Phase-Dependent Traffic Condition

We now develop a phase-based model for dynamic traffic conditions. This idea is consistent with both the classical two-phase (i.e., free flow and congested traffic) traffic theory and the more recent Kerner's three-phase (free flow, synchronized flow, and wide moving jam) theory [30], [69]. We make the following three observations on the real-world traffic as illustrated in Fig. 2:

- the whole duration of a day can be partitioned into different periods with stationary traffic conditions;
- while the traffic condition of a road segment may vary across different periods, it remains stationary in the same period for certain duration, each with a length of several hours;
- different road segments in the same time zone may have different traffic conditions, but the transitions between different traffic conditions are pretty synchronous.

We define traffic phases as follows:

**Definition 1** (Phase [30], [69]). *A (traffic) phase is a time interval during which the traffic conditions are stationary with fixed speed ranges for all road segments.*

There are multiple conceivable methods to construct phases using real-world data. We describe one based on speed clustering in Sec. VIII. The traffic conditions of the three road segments shown in Fig. 2 can be decomposed into six phases. We also observe that the intra-phase speed variances are substantially smaller than inter-phase speed variances. This observation justifies the use of phases as the first-order model for dynamic traffic conditions.

We now formulate PSPV under the setting of phase-dependent speed ranges. Let $t_0$ be the time for the truck to leave the origin $s$. Suppose the time interval $(t_0, t_0+T)$ can be divided into $N$ phases [3], i.e.,

$$(t_0, t_0+T) = \bigcup_{i\in\{1,2,...,N\}} \left( t_0 + \sum_{j=0}^{i-1} T_j, t_0 + \sum_{j=0}^{i} T_j \right), \tag{6}$$

where phase $i$ starts at time $t_0 + \sum_{j=0}^{i-1} T_j$ and ends at the time $t_0 + \sum_{j=0}^{i} T_j$. It is clear that $T_i$ is the length of phase $i$ and $\sum_{i=1}^{N} T_i = T$; we set $T_0 = 0$ for convenience of expression. For each $i = 1,2,...,N$, we denote the fixed minimum speed limit (resp. fixed maximum speed limit) of a road segment $e \in E_s$ at the phase $i$ by $R_e^{lb,i}$ (resp. $R_e^{ub,i}$), i.e.,

$$R_e^{lb,i} \;=\; R_e^{lb}\left(t_0 + \sum_{j=0}^{i-1} T_j\right), \; R_e^{ub,i} \;=\; R_e^{ub}\left(t_0 + \sum_{j=0}^{i-1} T_j\right). \tag{7}$$

We denote the edge sequence of a path $p$ by $\langle e_1, e_2, ..., e_{|p|}\rangle$, where $e_k \in E$ is an edge on $p$ for each $k = 1,2,...,|p|$ and we let $|p|$ denote the number of edges on $p$. We can map the edge $e_k \in p$ and $e_k \in E_s$ to a phase $i \in \{1,2,...,N\}$, once given a $p \in \mathcal{P}$ and a $\vec{t} \in \mathcal{T}_p$: for each $k = 1,2,...,|p|$,

$$I_{p,\vec{t}}(e_k) = i, \text{ if } e_k \in E_s \text{ and } \sum_{j=1}^{i-1} T_j \le \sum_{j=1}^{k-1} t_{e_j} < \sum_{j=1}^{i} T_j. \tag{8}$$

Given phased speed ranges, we can simplify the feasible set $\mathcal{T}_p$ in (3) to $\mathcal{T}_p^{\text{phase}}$ as follows:

$$\mathcal{T}_p^{\text{phase}} \triangleq \Big\{ \vec{t} : R_e^{lb,I_{p,\vec{t}}(e)} \le D_e/t_e \le R_e^{ub,I_{p,\vec{t}}(e)}, \quad \forall e \in E_s : e \in p \;;\; 0 \le t_e \le T, \forall e \in E_r : e \in p \Big\}. \tag{9}$$

Now PSPV under phased speed ranges can be formulated as:

$$\min_{p\in\mathcal{P},\vec{t}\in\mathcal{T}_p^{\text{phase}}} \quad \sum_{e\in p} c_e(t_e), \tag{10a}$$

$$\text{s.t.} \quad \sum_{e\in p} t_e \;\le\; T. \tag{10b}$$

The problem, while simplified, is still NP-hard as it covers the NP-hard problem PASO [19] as a special case.

**Proposition 2.** *PSPV under phased speed ranges is NP-hard.*

## V. Phase-Expanded Network and Phase-based Reformulation

To solve PSPV under phased speed ranges (i.e., the problem in (10)), we construct a phase-expanded network and use it

[3] Here we remark that the phase division depends on both the real-world traffic condition and the time window $(t_0, t_0+T)$. If $T$ is small, it's possible that only a fraction of a particular phase is covered. If $T$ is large, the time window may cover multiple phases.

to reformulate PSPV in this section. Our expanded-network-based formulation can be solved efficiently using a dual-subgradient-based algorithm to be introduced later in Sec. VI.

We construct the phase-expanded network $\tilde{G}(\tilde{V}, \tilde{E})$ from the input network $G(V, E)$ as follows: we define $\tilde{V} = \{v_i : \forall v \in V, \forall i \in [N]\}$ where $[N] \triangleq \{1, 2, 3, ..., N\}$ and $N$ is the number of phases. We let $\tilde{E} = H \cup L \cup R$, where $H$, $L$, and $R$ are sets of edges defined below, assuming $V_r = \{\mathsf{head}(e) : \forall e \in E_r\} \cup \{d\}$:

$$H \triangleq \cup_{i\in[N]} H_i, \text{ where } H_i = \{(u_i, v_i) : \forall (u, v) \in E\}, \quad (11)$$

$$L \triangleq \cup_{i\in[N-1]} L_i, \text{ where } L_i = \{(v_i, v_{i+1}) : \forall v \in V \backslash V_r\}, \quad (12)$$

$$R \triangleq \cup_{i\in[N-1]} R_i, \text{ where } R_i = \{(v_i, v_{i+1}) : \forall v \in V_r\}. \quad (13)$$

Here $v_i \in \tilde{V}$ denotes the node $v \in V$ in phase $i$, $(u_i, v_i) \in H_i \subseteq H$ denotes the edge $(u, v) \in E$ in phase $i$, and $(v_i, v_{i+1}) \in L_i \subseteq L$ (resp. $(v_i, v_{i+1}) \in R_i \subseteq R$) represents that a node $v \in V \backslash V_r$ (resp. a node $v \in V_r$) leaves phase $i$ and enters phase $i + 1$.

We denote the minimum travel time (resp. maximum travel time) of an edge $\tilde{e} \in \tilde{E}$ by $t^{lb}_{\tilde{e}}$ (resp. $t^{ub}_{\tilde{e}}$). According to the respective definitions of $H$, $L$, and $R$, we should have:

$$t^{lb}_{\tilde{e}} = \begin{cases} D_e/R^{ub,i}_e, & \text{if } \tilde{e} = (u_i, v_i) \in H \text{ and } e = (u, v) \in E_s, \\ 0, & \text{if } \tilde{e} = (u_i, v_i) \in H \text{ and } e = (u, v) \in E_r, \\ 0, & \text{if } \tilde{e} \in L \cup R, \end{cases} \quad (14)$$

$$t^{ub}_{\tilde{e}} = \begin{cases} D_e/R^{lb,i}_e, & \text{if } \tilde{e} = (u_i, v_i) \in H \text{ and } e = (u, v) \in E_s, \\ T_i, & \text{if } \tilde{e} = (u_i, v_i) \in H \text{ and } e = (u, v) \in E_r, \\ 0, & \text{if } \tilde{e} \in L, \\ T_i, & \text{if } \tilde{e} \in R. \end{cases} \quad (15)$$

For any $v \in V_r$, because one is allowed to wait at $v$ (either wait on the waiting edge $e \in E_r$ where $\mathsf{head}(e) = v$ that corresponds to a rest area or wait at the destination $d$ after arriving ahead of deadline), we set the maximum travel time from $v_i$ to $v_{i+1}$ to be $T_i$, i.e., one can wait at $v$ before leaving phase $i$ for at most time $T_i$ and enter phase $i + 1$; otherwise for any $u \in V \backslash V_r$, we set the maximum travel time from $u_i$ to $u_{i+1}$ to be 0, because waiting on actual road segments are not allowed and one cannot leave phase $i$ and enter phase $i+1$ if the arrival time at $u$ does not belong to the phase $i + 1$.

For each $\tilde{e} \in \tilde{E}$, it has the following fuel consumption function:

$$c_{\tilde{e}}(t) = \begin{cases} c_e(t), & \text{if } \tilde{e} = (u_i, v_i) \in H \text{ and } e = (u, v) \in E, \\ 0, & \text{if } \tilde{e} \in L \cup R. \end{cases} \quad (16)$$

We denote $E_i = H_i \cup L_i \cup R_i$, assuming $L_N = R_N = \emptyset$. We add an edge $(s, s_1)$ to $E_1$, and add an edge $(d_N, d)$ to $E_N$. For $(s, s_1)$ (resp. $(d_N, d)$), we set its maximum travel time to be 0 (resp. $T_N$), and set both its minimum travel time and fuel consumption to be 0. Fig. 4 illustrates our phase-expanded network, where a feasible solution to the problem (10) can be mapped to a corresponding solution in the phase-expanded network.

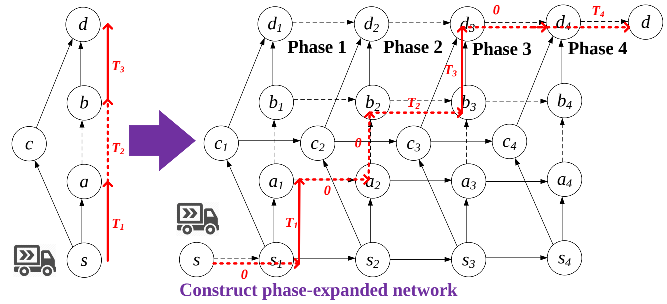



Fig. 4: An example of constructing a phase-expanded network $\tilde{G}$ from an input network $G$. For $G$, we assume that $(a, b)$ is a waiting edge that represents a rest area in $E_r$, while all the other edges represent road segments in $E_s$. Suppose the origin is $s$, the destination is $d$, and the deadline is $\sum_{i=1}^{4} T_i$ where $T_i$ is the length of phase $i$. Consider one solution in $G$ with a path of $\langle s, a, b, d\rangle$ and travel time assignments of $\{t_{sa} = T_1, t_{ab} = T_2, t_{bd} = T_3\}$. This solution represents that the truck travels from $s$ to $a$ with a driving time of $T_1$, then waits from $a$ to $b$ with a time of $T_2$, and finally drives from $b$ to $d$ with a driving time of $T_3$. It corresponds to a solution in $\tilde{G}$ with a path of $\langle s, s_1, a_1, a_2, b_2, b_3, d_3, d_4, d\rangle$ and travel time assignments of $\{t_{ss_1} = 0, t_{s_1a_1} = T_1, t_{a_1a_2} = 0, t_{a_2b_2} = 0, t_{b_2b_3} = T_2, t_{b_3d_3} = T_3, t_{d_3d_4} = 0, t_{d_4d} = T_4\}$.

In the following we formulate our problem PSPV under phased speed ranges using the phase-expanded network as follows:

$$\min_{\vec{x}\in\mathcal{X}, \vec{t}\in\mathcal{T}} \quad \sum_{\tilde{e}\in\tilde{E}} x_{\tilde{e}} \cdot c_{\tilde{e}}(t_{\tilde{e}}), \quad (17a)$$

$$\text{s.t.} \quad \sum_{\tilde{e}\in E_i} x_{\tilde{e}} \cdot t_{\tilde{e}} = T_i, \forall i \in [N], \quad (17b)$$

where $t_{\tilde{e}}$ defines the driving time (resp. waiting time) on $\tilde{e}$ if $\tilde{e}$ represents a road segment (resp. a rest area or the rest edge going into the destination). Since $\sum_{i\in[N]} T_i = T$, the deadline constraint is naturally guaranteed in our formulation 17. Note that we allow the truck to arrive ahead of the deadline by adding a rest edge next to the destination. We define $\mathcal{X}$ as the set of feasible paths from $s$ to $d$ in the phase-expanded network,

$$\mathcal{X} \triangleq \Big\{ \vec{x} : x_{\tilde{e}} \in \{1, 0\}, \forall \tilde{e} \in \tilde{E}, \text{ and} \\ \sum_{\tilde{e}\in\mathsf{Out}(\tilde{v})} x_{\tilde{e}} - \sum_{\tilde{e}\in\mathsf{In}(\tilde{v})} x_{\tilde{e}} = \mathbf{1}_{\tilde{v}=s} - \mathbf{1}_{\tilde{v}=d}, \forall \tilde{v} \in \tilde{V} \Big\}, \quad (18)$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function, which takes the value 1 (resp. 0) if the statement in $\{\cdot\}$ is true (resp. false), and $\mathsf{Out}(\tilde{v}) = \{(\tilde{v}, \tilde{u}) : \forall (\tilde{v}, \tilde{u}) \in \tilde{E}\}$ (resp. $\mathsf{In}(\tilde{v}) = \{(\tilde{u}, \tilde{v}) : \forall (\tilde{u}, \tilde{v}) \in \tilde{E}\}$) is the set of outgoing edges (resp. incoming edges) of node $\tilde{v} \in \tilde{V}$. Define

$$\mathcal{T} \triangleq \{\vec{t} : t^{lb}_{\tilde{e}} \le t_{\tilde{e}} \le t^{ub}_{\tilde{e}}, \forall \tilde{e} \in \tilde{E}\} \quad (19)$$

as the set of possible travel times of traversing each edge $\tilde{e} \in \tilde{E}$. The formulation (17) based on phase-expanded network simplifies the PSPV problem shown in (2). However, the problem in 17 is still a mixed-integer nonlinear programming problem with nonlinear fuel cost function $c_{\tilde{e}}(t_{\tilde{e}})$ and decision variables $t_{\tilde{e}}$ and $x_{\tilde{e}}$ multiplying together.

In the formulation in (17), the objective in (17a) minimizes the total fuel consumption and the time-aware constraints in (17b) restrict that the aggregate travel time (recall that travel time includes both driving time, waiting time before arriving at the destination, and the waiting time at the destination if the truck arrives ahead of deadline) in each phase is equal to the length of the phase. Further, since our phase-expanded network includes edges $(d_i, d_{i+1})$ with a travel time upper bound of $T_i$, for each $i = 1, 2, ..., N-1$, our formulation's feasible set naturally includes solutions of arriving at the destination $d$ ahead of deadline. In addition, the total travel time in a phase should be equal to the length of this phase. We remark that for (17b) we use equality constraints instead of inequality constraints, because we assume that the phase does not change while the truck is running on any edge $e \in E_s$ and hence can only change when the truck switches from one edge to another or waiting on a virtual rest edge $e \in E_r$. Our assumption comes from a practical consideration that the length of a phase is substantially longer than the travel time on a road segment (several hours vs. several minutes.). Meanwhile, such equality constraint simplifies our model by ruling out the case of driving in two consecutive phases on the same road segment. Therefore, it is reasonable to use equality constraints so that we can map a solution in the expanded graph $\tilde{G}$ to a solution in the actual transportation network $G$. We also remark that departure time can be taken into account as a decision variable by inserting a (pseudo) rest edge going out from the origin. Then choosing later departure time can be modelled as waiting on the rest edge.

## VI. A Subgradient-based algorithm for PSPV under phased speed ranges

In this section, we develop an efficient dual-subgradient based algorithm for PSPV under phased speed ranges.

### A. The Dual Problem

A critical observation of the problem in (17) is that we can decompose the dual problem to convex optimization problems and a shortest path problem, once given specific dual variables. Specifically, we first relax the time-sensitive constraints in (17b), and get the following Lagrangian function:

$$\begin{aligned}\mathcal{L}(\vec{x}, \vec{t}, \vec{\mu}) &= \sum_{i\in[N]} \sum_{\tilde{e}\in E_i} x_{\tilde{e}} c_{\tilde{e}}(t_{\tilde{e}}) + \sum_{i\in[N]} \mu_i (\sum_{\tilde{e}\in E_i} x_{\tilde{e}} t_{\tilde{e}} - T_i) \\ &= \sum_{i\in[N]} \sum_{\tilde{e}\in E_i} x_{\tilde{e}} \cdot (c_{\tilde{e}}(t_{\tilde{e}}) + \mu_i t_{\tilde{e}}) - \sum_{i\in[N]} \mu_i T_i.\end{aligned} \tag{20}$$

Then corresponding dual function and dual problem are

$$\max_{\vec{\mu}} \mathcal{D}(\vec{\mu}) \triangleq \max_{\vec{\mu}} \min_{\vec{x}\in\mathcal{X}, \vec{t}\in\mathcal{T}} \mathcal{L}(\vec{x}, \vec{t}, \vec{\mu}). \tag{21}$$

We observe that

$$\begin{aligned}&\mathcal{D}(\vec{\mu}) \\ =& -\sum_{i\in[N]} \mu_i T_i + \min_{\vec{x}\in\mathcal{X}, \vec{t}\in\mathcal{T}} \sum_{i\in[N]} \sum_{\tilde{e}\in E_i} x_{\tilde{e}} \cdot (c_{\tilde{e}}(t_{\tilde{e}}) + \mu_i t_{\tilde{e}}) \\ =& -\sum_{i\in[N]} \mu_i T_i + \min_{\vec{x}\in\mathcal{X}} \sum_{i\in[N]} \sum_{\tilde{e}\in E_i} x_{\tilde{e}} \cdot \min_{t_{\tilde{e}}^{lb} \le t_{\tilde{e}} \le t_{\tilde{e}}^{ub}} (c_{\tilde{e}}(t_{\tilde{e}}) + \mu_i t_{\tilde{e}}) \\ =& -\sum_{i\in[N]} \mu_i T_i + \min_{\vec{x}\in\mathcal{X}} \sum_{\tilde{e}\in\tilde{E}} x_{\tilde{e}} \cdot w_{\tilde{e}}(\vec{\mu}),\end{aligned} \tag{22}$$

where

$$w_{\tilde{e}}(\vec{\mu}) = c_{\tilde{e}}(t_{\tilde{e}}^*(\mu_i)) + \mu_i t_{\tilde{e}}^*(\mu_i) \tag{23}$$

represents a combined cost of fuel and time and $t_{\tilde{e}}^*(\mu_i)$ is the optimal solution of the cost minimization problem with each edge $\tilde{e} \in E_i$, for all $i \in [N]$. Let $p^*(\vec{\mu})$ be the corresponding shortest path from $s$ to $d$ in the phase-expanded network. We have

$$\mathcal{D}(\vec{\mu}) = -\sum_{i\in[N]} \mu_i T_i + \sum_{\tilde{e}\in p^*(\vec{\mu})} w_{\tilde{e}}(\vec{\mu}). \tag{24}$$

Essentially, given specific dual variables $\vec{\mu}$, we can solve a shortest path problem in the phase-expanded network to obtain the value of the dual function $\mathcal{D}(\vec{\mu})$.

### B. A Dual-Subgradient Algorithm

It is straightforward to verify that the subgradient of the dual function with respect to each $\mu_i \in \vec{\mu}$ is given by $\delta_i(\vec{\mu}) - T_i$, where $\delta_i(\vec{\mu})$ is the aggregate travel time of edges $\tilde{e} \in E_i$ and $\tilde{e} \in p^*(\vec{\mu})$ defined as follows:

$$\delta_i(\vec{\mu}) \triangleq \sum_{\tilde{e}\in p^*(\vec{\mu}) \text{ and } \tilde{e}\in E_i} t_{\tilde{e}}^*(\mu_i). \tag{25}$$

Intuitively, the dual variable $\mu_i$ can be interpreted as the delay price in phase $i$. Larger delay price results in less travel time in the phase, as formalized in the following lemma.

**Lemma 1.** *$\delta_i(\vec{\mu})$ is non-increasing in $\mu_i$, for all $i \in [N]$.*

*Proof.* We refer the interested reader to Appendix A. □

The dual-subgradient algorithm iteratively updates the dual variables $\vec{\mu}$ according to their subgradients. In the $k$-th iteration ($k \ge 1$),

$$\mu_i(k) = \mu_i(k-1) + \phi(k) \cdot \left[\delta_i(\vec{\mu}(k-1)) - T_i\right], \ \forall i \in [N], \tag{26}$$

where $\phi$(k) is the step size to be discussed later. When the dual variable is small and the corresponding phase duration constraint is violated, the corresponding sub-gradient would be positive. The algorithm will then increase the dual variable, i.e., the delay price, and consequently the travel time in the corresponding phase will decrease, aiming to meet the phase duration constraint. When the phase duration constraints are not violated, the corresponding sub-gradients would be negative and the dual variable would decrease, giving more weight to the fuel cost in the combined edge cost to optimize when computing the dual function in (24). We summarize the dual-subgradient algorithm in Algorithm 1. We define sol as the solution corresponding to the dual variable $\vec{\mu}$ as the

**Algorithm 1** A Dual-subgradient Algorithm for PSPV under Phased Speed Ranges

```
procedure
    Set sol = NULL, ite = 1, and μ_i = 0, ∀i ∈ [N].
    while ite ≤ ITE_LIMIT do
        ite = ite+1
        Compute the solution sol′ corresponding to μ⃗.
        if δ_i(μ⃗) − T_i = 0, ∀i ∈ [N] then
            return sol ← sol′
        if sol′ is feasible and consumes less fuel compared to sol then
            sol ← sol′
        Update μ⃗ according to (26), ∀i ∈ [N]
    return sol
```

path $p^*(\vec{\mu})$ with a travel time of $t^*_{\tilde{e}}(\mu_i)$ assigned to each edge $\tilde{e} \in E_i$, for all $i \in [N]$.

It is known the subgradient methods always converges, at a rate of $O\left(1/\sqrt{k}\right)$ [70]. Specific to our scenario, we have the following observation.

**Theorem 1** (Dual Value Convergence)**.** *Let $\bar{\mathcal{D}}_k$ be the best dual value observed in the first $k$ iterations of Alg. 1. Let $\mathcal{D}^*$ be the optimal dual value, there exists a constant step size, i.e., $\phi(\cdot) = \phi > 0$, and a constant $C > 0$ such that*

$$\mathcal{D}^* - \bar{\mathcal{D}}_k \leq \frac{C}{\sqrt{k}}, \quad \forall k \geq 1. \tag{27}$$

*Proof.* The proof is presented in Appendix B. □

Thm. 1 shows that we can achieve $O(1/\sqrt{k})$ convergence rate by properly choosing a constant step size. However, the convergence may not be satisfactory in practice. Therefore, we adaptively tune the step size according to the method proposed in [71]. We first initialize a large step size. Gradually, we decrease it as the gap between the dual value and recovered primal value gets smaller. When we detect that the gap is smaller than a predefined threshold, we decrease the step size even faster. In this way, one is expected to achieve an improved convergence speed in practice [71].

However, $p^*(\vec{\mu})$ and $t^*_{\tilde{e}}(\vec{\mu})$, $\tilde{e} \in \tilde{E}$ do not necessarily form a feasible primal solution. In such cases, we fix the corresponding path in the original highway network and the driving time on actual road segments. And then we re-optimize the waiting time and re-index the phase to try to recover a feasible primal solution.

In the following, we further give a set of complementary-slackness-like conditions under which the solution returned by our algorithm is optimal to our PSPV problem under phased speed ranges.

**Theorem 2.** *If dual variables $\vec{\mu}$ satisfy*

$$\delta_i(\vec{\mu}) - T_i \ = \ 0, \quad \forall i \in [N], \tag{28}$$

*then the corresponding $p^*(\vec{\mu})$ with a travel time of $t^*_{\tilde{e}}(\mu_i)$ assigned to each edge $\tilde{e} \in E_i$, for all $i \in [N]$, is an optimal solution to our PSPV problem under phased speed ranges.*

*Proof.* The proof is presented in Appendix C. □

Our PSPV problem under phased speed range is NP-hard (Lem. 2), which implies that we can not design an algorithm to find the optimal solution in polynomial time unless P = NP. However, Thm. 2 gives a sufficient condition to certify the optimality of our solution. The theorem implies that for some special instances, optimal solution is attainable using our dual-subgradient based algorithm. The sufficient condition essentially means that the phase duration constraint is strictly satisfied in all the phases, thus leaving no space for further speed optimization and opportunistic driving. However, such equality constraints are not easy to satisfy in our experiment introduced later due to the inherent structure of our problem. Nevertheless, we find our solutions still achieve great performance that is much better than the shortest path based alternative (See Sec. VIII.).

Overall, we develop a dual-subgradient algorithm to solve large-/national- scale PSPV problem instances under phased speed ranges. We further give conditions under which our algorithm outputs an optimal solution.

## VII. Discussions

### A. Impact of opportunistic driving on traffic condition

Intuitively, the traffic condition may be affected if a large number of trucks perform opportunistic driving (e.g., they all choose to drive during a lightly-congested period, and as a result, the traffic turns heavily congested, which in turn, may increase the fuel consumption). Today, such influences may be minor, since the number of trucks only constitutes $4\%$ of the total vehicle population. In the future, the population of trucks may grow and other vehicles may also carry out opportunistic driving. It remains an interesting research direction to consider the influence on traffic condition when a substantial number of vehicles, including trucks, perform opportunistic driving.

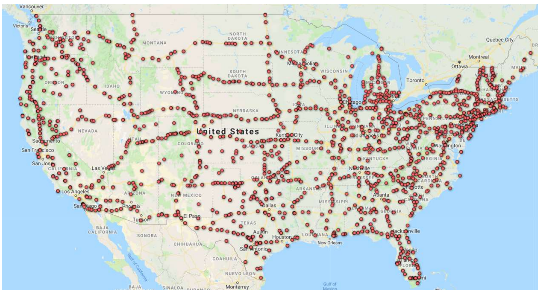
### B. Interactions of multiple vehicles

We focus on the case of a single truck to identify the design space of opportunistic driving, develop a model and scheme to extract the maximum benefit, and demonstrate the potential performance. As an important next step, it is of great interest to study the scenarios where multiple vehicles interplay strategically via opportunistic driving. The traffic condition may be affected if a large number of trucks perform opportunistic driving. For example, if they all choose to drive during a lightly-congested period, then the traffic turns heavily congested, which in turn, increases the fuel consumption.

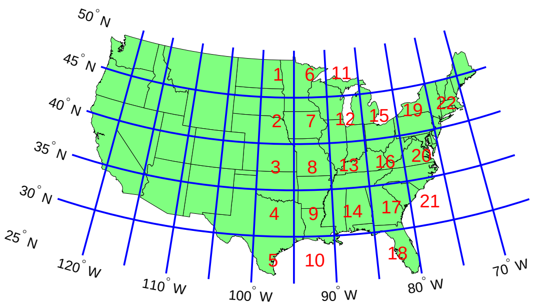
It remains an interesting research direction to analyze the strategic interactions among multiple vehicles, performing opportunistic driving, including trucks and other types of vehicles.

### C. Robustness to real-world speed perturbation

The phased speed range is a first-order model of the dynamic traffic speed. In practice, speed range may be perturbed by random factors such as weather. However, such perturbation is on a small scale and would not have a significant effect on



(a)




(b)

Fig. 5: Simulated US national highway network. (a) Rest areas in the US national highway network (with red marks) [14]. (b) Simulated US regions (with red numbers) [18].

the overall fuel consumption. We verify it in our simulation using real-world traffic speed trace. We use a simple "re-balance" heuristic that we round our assigned speed to real speed range if the perturbed maximum speed limit is even smaller than the assigned speed and rest less or drive faster if we are behind the time schedule in the original solution. From the simulations results in Sec. VIII, we observe that only less than $1\%$ of the feasible instances miss the deadline, with an average deadline violation percentage of $0.6\%$. Meanwhile, the increase in fuel consumption due to real-world speed range perturbation is only $0.8\%$. Hence, our solution is robust to real-world speed perturbation.

### *D. Computation and real-time traffic condition*

In this paper, we focus on computing an energy-efficient path plan, speed plan, and opportunistic driving plan for long-haul heavy-duty truck. Ideally, the computation is done before the truck starts the trip and the truck will follow the optimized driving profile to minimize the fuel consumption. In practice, however, real-time traffic conditions may deviate from those used in the pre-departure optimization, thus the truck may need to adjust the driving profiles on the fly to continuously pursue optimized fuel efficiency. For example, to cope with the real-time traffic variations, we can re-optimize path planning, speed planning, and opportunistic driving repeatedly at regular intervals with real-time traffic information [72], e.g., once every 15 or 30 minutes.

## VIII. Performance Evaluation

We implement our solutions using C++ and python. We run the experiments on a server cluster with $42$ pieces of 2.4 GHz - 3.4 GHz Intel/AMD processors, each equipped with 20GB memory on average. We use real-world traces to evaluate the performance of our algorithm. We represent a PSPV instance by a tuple of $(s, d, T, t_0)$, where $s$ is the origin, $d$ is the destination, $T$ is the deadline, and $t_0$ is the truck departure time.

*Transportation network and heavy-duty truck*. We construct the US national highway network using the data from the Clinched Highway Mapping Project [73]. We merge consecutive road segments with the same grade. We then focus on its eastern part with $38,213$ connecting points and $82,781$ directed road segments. As illustrated in Fig. 5b, we divide the eastern US into 22 regions.Our simulated truck is a class-8 heavy-duty truck Kenworth T800, which is fully loaded with 36-ton cargo [74].

*Rest edges*. We collected the location information of rest areas in the US from POI (point of interest) Factory [75]. We insert each rest area into the closest road segment of the US highway network as a rest edge. Fig. 5a illustrates the distribution of rest areas over the US highway network. We find $1,906$ rest areas in total with truck parking lots in the highway network of the eastern US, which corresponds to a rest area density of $\frac{1906}{82781} = 0.023$.

*Variable speed ranges*. To model road speed ranges that are dependent on the dynamic traffic condition, we collect real-time road speed data from HERE map [22] for ten days $(08/18/2017 - 08/27/2017)$.

*Constructing Phases*. We first divide one day into $48$ time slots with each time slot's length to be $30$ minutes. For time slot $t$, we can get a speed vector $\mathbf{v}_t := (v_t^e)_{e\in E}$. Here $v_t^e$ is the average speed on road segment $e$ at time slot $t$. The vector $\mathbf{v}_t$ characterizes the traffic condition over the simulated highway network at time slot $t$. We then use $\mathbf{v}_t$ as a feature vector and apply the K-means algorithm [76] to group different time slots. For each phase, we use the average road traffic speed as the road maximum speed limit in this phase. We set the minimum road speed limit to be the minimum of $15$mph and the average speed. We finally divide one day into the following 6 phases: (11 PM, 7 AM the next day), (7 AM, 8:30 AM), (8:30 AM, 3:30 PM), (3:30 PM, 6:30 PM), (6:30 PM, 8 PM) and (8 PM, 11 PM). The phase division coincides with our intuition that early morning (11 PM, 7 AM the next day) and mid-day should have different traffic conditions. There are also intermediate phases between them. Fig. 6 illustrates the phase construction result for the three randomly selected road segments in the eastern US. We can see that the phase construction results roughly coincide with the average speed dynamics on the three road segments.

*Road fuel consumption*. We first collect the grade of each road segment based on the elevations of nodes provided by the Elevation Point Query Service [77]. We then use the advanced vehicle simulator (ADVISOR) [78] to collect the fuel consumption rate data with different truck driving speeds for different road grades. Finally, we use MATLAB to fit the

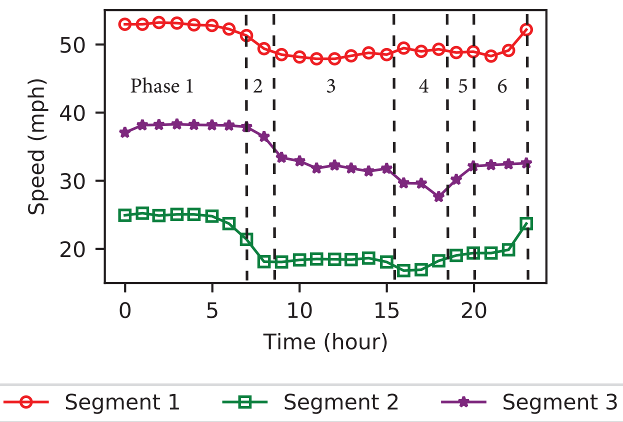



Fig. 6: The average traffic speeds across one day of three randomly selected road segments in the eastern US highway. The vertical lines divide one day into different phases.

fuel consumption rate function using a 3-order polynomial for each specific road grade. The same model of fuel consumption rate function was also used in related studies [18]–[20].

*Origin-destination pair*. We obtain the 2016 American truck freight transportation origin-destination statistics from [79]. We divide the eastern part of the US into 22 regions as illustrated in Fig. 5b. We focus on the cross-region long-haul truck trip and sample the busiest trips' origin-destination region pairs. We manually set the origin (resp. destination) node to the nearest one to the center of the representative origin (resp. destination) region.

### A. Simultaneous Reduction on both Fuel Consumption and Driving Time by Opportunistic Driving

As discussed in Sec. III-A and illustrated by Fig. 3, the fuel consumption of traversing a road segment is a convex function of the driving speed in a specific range. The function first decreases and then increases with the driving speed. In the decreasing part, increasing speed benefits both fuel saving and driving time reduction, while in the increasing part, fuel saving is at the expense of lowering speed and increasing driving time. Our solution allows a truck to opportunistically traverse busy road segments at a more fuel-efficient speed. Therefore, the truck may be able to save both driving time and fuel consumption of traversing such road segments. In contrast, the existing solution PASO [18], [19] assumes static speed ranges and only saves fuel at the expense of increasing driving hours.

A conceivable approach that generalizes PASO to our setting is the following. We first consider a static-speed-range-setting where we set the minimum speed limit (resp. maximum speed limit) to be the weighted average minimum speed limit (resp. average maximum speed limit) in all phases with phase length as the weight. Thus we get an instance of PASO that can be solved by the existing algorithm from [18]. We then round the solution in that if the driving speed exceeds the variable road speed range, we reset it to be the maximum variable speed limit in the corresponding phase. After running the

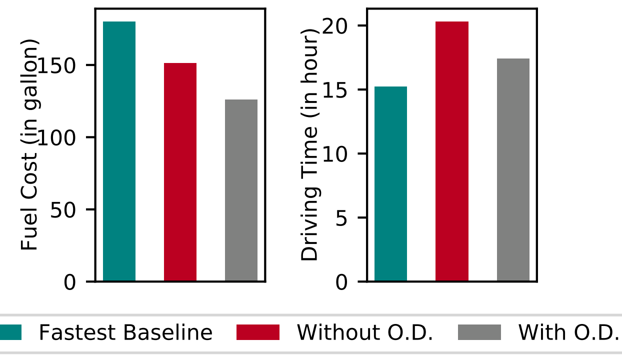



Fig. 7: We set $s = 14$, $d = 15$, $t_0 = 5$ AM and $T = 25.0h$. Our solution with O.D. achieves about $10\%$ more fuel saving than optimization without O.D. while incurring less driving time. The reason is that the solution with O.D. does several hours' strategic rest at rest areas during the phase 3 (8:30 AM, 3:30 PM) and the phase 6 (8PM, 11PM), which helps truck avoid driving in congestions in urban areas. In contrast, solutions without opportunistic driving does not exploit such design space and has to drive in unfavorable traffic conditions with low fuel efficiency, thus incurring higher fuel consumption. This set of results highlight that opportunistic driving not only saves fuel but also reduces driving time as compared to solutions without opportunistic driving.

experiments, we observe that in $22\%$ of the instances, the truck fails to arrive at the destination on time when we apply the conceivable approach.

Fig. 7 gives simulation results that suggest that our solution significantly reduces both fuel consumption and driving time as compared to optimization without opportunistic driving. In some instances, we observe that the driving time with opportunistic driving is even shorter than that of driving as fast as possible on the shortest path. For example, when we set $s = 9, d = 12$, the total driving time of our solution is $19.4$ hours, while the total driving time of the fastest solution without opportunistic driving is $22.1$ hours. The reason is that at the departure time, the traffic condition is bad, and the truck could not drive at a high speed even if it drives as fast as possible. With opportunistic driving, however, the truck is allowed to strategically wait and avoid the bad traffic condition. This example highlights the possibility that opportunistic driving can save driving time even compared to driving as fast as possible without opportunistic driving.

Fig. 8 gives an example of our route planning results. In this example, the solution with opportunistic driving consumes $133.5$ gallons of fuel, which is $21\%$ less than that of the solution without opportunistic driving ($169.0$ gallons). Without opportunistic driving, the driving time and thus the total travel time is $25.7$h for the truck. With opportunistic driving, the driving time reduces to $21.3$h. With $8.7$h strategic waiting time, the total travel time with opportunistic driving is $30.0$h. We observe that opportunistic driving reduces the driving time by $17\%$, without violating the total travel time deadline. Intuitively, the savings of both fuel consumption and driving time are from driving in favorable traffic conditions, thanks to

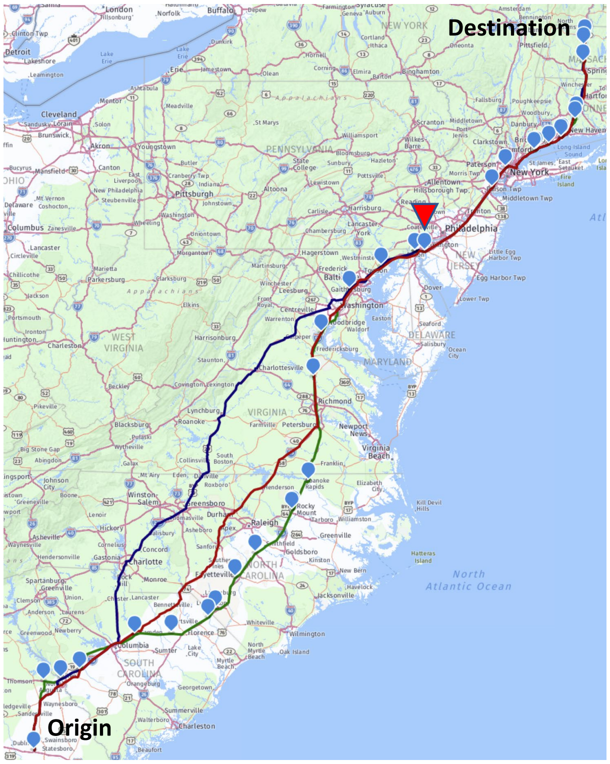



Fig. 8: An example of route planning results, where the truck travels from the origin in Georgia to the destination in Massachusetts. The dark blue path is the shortest path. The red path is the energy-efficient timely transportation solution without opportunistic driving. The green path is our solution with opportunistic driving. The light-blue droplets represent the rest areas along the path of our solution. We note that our solution waits strategically at the rest area located around I-95, Newark, DE 19713 (N39°41′, W75°39′) marked by a red triangle in the figure, for about five hours, before entering the metropolitan area of Philadelphia, to avoid the congestion in the morning.

opportunistic driving.

### B. Performance Evaluation of Our Algorithm

We conduct extensive simulations to evaluate our solution. For the shortest-path based alternative, we drive as fast as possible along the shortest path.[4] We sample 25 different $(s, d)$ pairs that evenly cover the eastern US. Given an $(s, d)$ pair, we select an hour of the day uniformly at random, and a deadline $T$ from $1.3 \cdot T_{\min}$ to $2.0 \cdot T_{\min}$ with a step of $0.1 \cdot T_{\min}$ ($T_{\min}$ is the minimum travelling time from origin to destination under average static speed ranges). We simulate a total of $840$ feasible instances and present the simulation results on average in Tab. II. We observe that our solution saves $20\%$ fuel as compared to the fastest-/shortest- path baselines, where

[4]To generate the shortest path, we tried weights including distance, minimum travel time and fuel cost at the fastest feasible speed under static speed range. Then we drive as fast as possible on the paths under dynamic traffic conditions. We find that the fuel costs incurred on the shortest paths with different weights are very close to each other. The differences are within $0.5\%$. Therefore, we choose distance as the representative weight.

| | Shortest Path Baseline | Our Solution (without O.D.) | Our Solution (with O.D.) |
|---|---|---|---|
| **Fuel (gallon)** | 114.4 | 105.9 | 91.6 |
| **Time (hour)** | 14.0 | 16.2 | 15.1 |

TABLE II: Fuel consumption and driving time performance of different algorithms.

$13\%$ is contributed by opportunistic driving. Furthermore, our solution with opportunistic driving saves up to $6\%$ driving time as compared to solutions without opportunistic driving.

We highlight that the benefit of opportunistic driving is two-fold. First, it allows one to efficiently save fuel. Second, it, may at the same time, reduce driving time. Moreover, trucks implementing opportunistic driving are likely to avoid driving in heavy traffic conditions, thus alleviating traffic congestion. Therefore, it is a *win-win-win* strategy.

### C. Impact of Truck Departure Time and Deadline

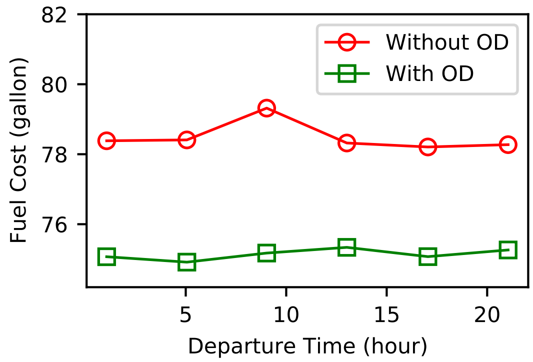
It is intuitive that one can reduce fuel consumption if the truck leaves the origin at a time without traffic congestion. We now study the impact of the departure time $t_0$ on the fuel consumption.

Fig. 9a gives the fuel consumption of different $t_0$, with $s = 16$, $d = 17$, and $T = 17$ hours. As seen in Fig. 9a, opportunistic driving saves $4\%$ fuel on average in this instance, which is considered significant for heavy-duty truck operation. We also observe that there is a morning fuel consumption peak, which coincides with our intuition that driving in the morning usually consumes more fuel, when the traffic is congested. Besides, we again observe that opportunistic driving can save fuel, where the fuel consumption of our solution with opportunistic driving is less than that without opportunistic driving. We also observe that perhaps interestingly, the fuel consumption peak phenomenon is relieved when we allow opportunistic driving. The reason is that we can strategically wait and opportunistically traverse those congested roads off the rush hour. Next, we show the impact of the input deadline. Fig. 9b gives the fuel consumptions of our solution with different deadlines $T$, with $s = 16$, $d = 17$, and $t_0 = 19$ PM. With different deadlines, our solution with opportunistic driving consistently saves fuel as compared to that without opportunistic driving. On average, opportunistic driving saves $4\%$ fuel in this instance. We observe that fuel consumption with and without opportunistic driving both decrease a lot as deadline increases for the results without opportunistic driving. This is because larger deadlines allow more room for speed planning.

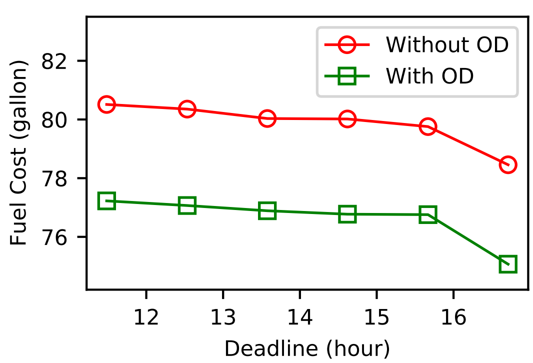
Overall, the above observations, again, verify that opportunistic driving saves more fuel than only optimizing path and speed. The performance gain does not come from merely driving slower but also from opportunistic driving.

### D. Dynamic vs. Static Traffic Conditions

In this subsection, we highlight a perhaps surprising observation that dynamic traffic conditions (and hence variable speed ranges) can offer even more fuel saving potential than




(a)




(b)

Fig. 9: Comparison of fuel consumption under different departure time and deadlines. (a) Fuel consumption of our solution for the instance of $s = 16$, $d = 17$, $T = 17$ hours, with different departure times $t_0$. (b) Fuel consumption of our solution for the instance of $s = 16$, $d = 17$, $t_0 = 7$ PM, with different deadlines $T$.

static traffic conditions. Opportunistic driving allows us to capitalize on such potential.

We set the static speed range of each road as the average of the variable speed ranges. We use the existing algorithm from [18], [19] to obtain the fuel consumption under the setting of static speed ranges. We run our algorithm to obtain the fuel consumption under the setting of variable speed ranges. Fig. 10 presents the fuel-consumption-ratio results averaged over 200 feasible instances. We notice that without opportunistic driving, the fuel consumed under dynamic speed range is consistently larger than that under static speed range. Meanwhile, opportunistic driving can allow us to exploit the variable speed ranges to achieve a much larger fuel saving than that under the static speed range, as much as $16\%$. Further, as the delay factor increases, with opportunistic driving, the fuel-saving ratio is strictly decreasing by as much as $3\%$. This is because a larger deadline allows larger design space for strategic waiting and speed planning, thus larger fuel saving. In contrast, without opportunistic driving, the ratio stays roughly the same. Fig. 11 shows that the rest time ratio, which is defined to be the ratio of rest time to total trip time, increases with the deadline. The ratio almost doubles from

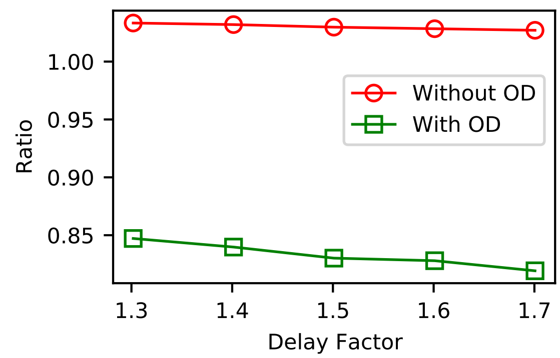



Fig. 10: Ratio comparing the fuel consumption of the setting of variable speed ranges to that of the setting of static speed ranges, with respect to the delay factor, which is defined as the ratio of input deadline to time cost of the fastest solution under static speed range. The smaller the ratio, the better the fuel saving performance as compared to that under static speed range.

$15\%$ to $28\%$, when we extend the deadline by $31\%$. The result highlights that extending the deadline allows more room for opportunistic driving. In particular, we see the percentage of strategic waiting time increases, which in turn contributes to larger fuel saving.

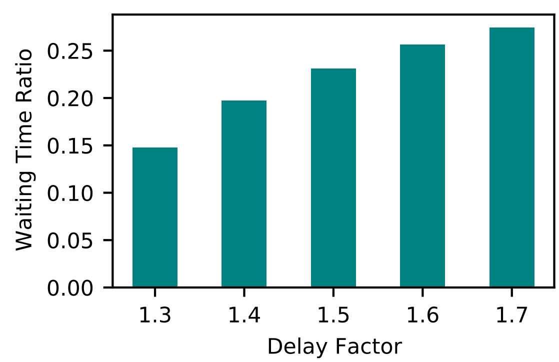



Fig. 11: The plot of the waiting time ratio as the delay factor varies. Here the waiting time ratio is defined as the ratio of the aggregate waiting duration over the total trip time.

## IX. Conclusion and Future Work

We study the problem of minimizing the fuel consumption of a heavy-duty truck traveling across national highway subject to a hard deadline, where traversing a road is subject to variable speed ranges due to dynamic traffic conditions. We propose a new design space, *opportunistic driving*, to exploit the dynamic traffic conditions for saving fuel. We observe that real-world speed ranges are mostly phase-dependent, where a phase is a time interval with stationary traffic conditions and hence fixed speed ranges. We prove that our problem under phased speed ranges is NP-hard. We then develop an efficient dual-subgradient algorithm for solving large-/national- scale instances. The algorithm always converges and we characterize conditions under which the algorithm generates an optimal solution. We use real-world traces to conduct extensive simulations over the US highway system, and observe that our algorithm saves up to $20\%$ fuel compared to fastest-/shortest- path baselines, among which $13\%$ is contributed by opportunistic driving. Meanwhile, opportunistic driving also reduces driving time by 6% as compared to only optimizing path planning and speed planning. As such, opportunistic driving offers a desirable design option to simultaneously reduce fuel consumption and hours of driving. We also observe that, perhaps surprisingly, dynamic traffic conditions can be exploited to save more fuel than that under static traffic conditions.

It is an interesting future direction to evaluate the impact of opportunistic driving on background traffic oblivious to opportunistic driving. It is also interesting to model and analyze the interactions among vehicles performing opportunistic driving.

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## APPENDIX A
## PROOF OF LEM. 1

*Proof.* We show this for $\delta_i$ and $\mu_i$ while keeping $\mu_{-i}$ fixed. Let us consider any two $\mu_{i1}$, $\mu_{i2}$ with $\mu_{i1} \leq \mu_{i2}$. Suppose the optimal path for $\mu_{i1}$ is $p_1$, and the optimal path for $\mu_{i2}$ is $p_2$. For any path $p$ and $\mu_i$, we denote its optimal generalized path cost by

$$W_p(\mu_i) = C(\mu_i) + \Delta_p(\vec{t}^*(\mu_i), \mu_i), \tag{29}$$

where we denote its augmented path fuel cost by

$$\Delta_p(\vec{t}, \mu_i) = \sum_{i\in[N]} \sum_{\tilde{e}\in E_i} x^*_{\tilde{e}}(\mu_i) C_{\tilde{e}}(\mu_i, t_{\tilde{e}}). \tag{30}$$

For simplicity here we omit the other dual variables in $C(\cdot)$ and $C_{\tilde{e}}(\cdot)$ since they are fixed. By the definition of $\Delta(\cdot)$, we have

$$\Delta_p(\vec{t}, \mu_{i1}) - \Delta_p(\vec{t}, \mu_{i2}) = (\mu_{i1} - \mu_{i2})(\delta_i(\vec{t}, p) - T_i), \tag{31}$$

where $\delta_i(\vec{t}, p) := \sum_{\tilde{e}\in E_i \cap p} t_{\tilde{e}}$. Since $p_1$ is the optimal path for dual problem when $\mu_i = \mu_{i1}$, using the optimality of $p_1$, we have

$$\begin{aligned} W_{p_1}(\mu_{i1}) = &\quad C(\mu_{i1}) + \Delta_{p_1}(\vec{t}^*(\mu_{i1}), \mu_{i1}) \\ \leq &\quad C(\mu_{i1}) + \Delta_{p_2}(\vec{t}^*(\mu_{i1}), \mu_{i1}) \\ \leq &\quad C(\mu_{i1}) + \Delta_{p_2}(\vec{t}^*(\mu_{i2}), \mu_{i1}). \end{aligned} \tag{32}$$

The second inequality is true because

$$\begin{aligned} C_{\tilde{e}}(\mu_{i1}, t^*_{\tilde{e}}(\mu_{i1})) = &\quad \min\nolimits_{t^l_{\tilde{e}} \leq t_{\tilde{e}} \leq t^u_{\tilde{e}}} C_{\tilde{e}}(\mu_{i1}, t_{\tilde{e}}) \\ \leq &\qquad C_{\tilde{e}}(\mu_{i1}, t^*_{\tilde{e}}(\mu_{i2})), \end{aligned} \tag{33}$$

where $C_{\tilde{e}}(\mu_i, t_{\tilde{e}}) := c_{\tilde{e}}(t_{\tilde{e}}) + \mu_j t_{\tilde{e}},\ \tilde{e} \in E_j$. Then, we can get

$$\Delta_{p_1}(\vec{t}^*(\mu_{i1}), \mu_{i1}) \leq \Delta_{p_2}(\vec{t}^*(\mu_{i2}), \mu_{i1}). \tag{34}$$

Similarly, for $p_2$ and $\mu_{i2}$, we can get

$$\begin{aligned} W_{p_2}(\mu_{i2}) = &\quad C(\mu_{i2}) + \Delta_{p_2}(\vec{t}^*(\mu_{i2}), \mu_{i2}) \\ \leq &\quad C(\mu_{i2}) + \Delta_{p_1}(\vec{t}^*(\mu_{i1}), \mu_{i2}), \end{aligned} \tag{35}$$

which is equivalent to

$$\Delta_{p_2}(\vec{t}^*(\mu_{i2}), \mu_{i2}) \le \Delta_{p_1}(\vec{t}^*(\mu_{i1}), \mu_{i2}). \tag{36}$$

Summing inequality 34 and inequality 36, we get that

$$\begin{aligned}\Delta_{p_1}(\vec{t}^*(\mu_{i1}), \mu_{i1}) - \Delta_{p_1}(\vec{t}^*(\mu_{i1}), \mu_{i2}) \le \\ \Delta_{p_2}(\vec{t}^*(\mu_{i2}), \mu_{i1}) - \Delta_{p_2}(\vec{t}^*(\mu_{i2}), \mu_{i2}).\end{aligned} \tag{37}$$

By the definition of $\Delta(\cdot)$, we can get

$$(\mu_{i1} - \mu_{i2})(\delta_i(\vec{t}^*(\mu_{i1}), p_1)) - \delta_i(\vec{t}^*(\mu_{i2}), p_2)) \le 0. \tag{38}$$

Since we assume that $\mu_{i1} \le \mu_{i2}$, we must have

$$\delta_i(\vec{t}^*(\mu_{i1}), p_1)) \ge \delta_i(\vec{t}^*(\mu_{i2}), p_2). \tag{39}$$

This means $\delta_i^*(\vec{\mu})$ is nonincreasing in $\mu_i$. □

## Appendix B
## Proof of Thm. 1

*Proof.* We follow the proof idea from the discussion in [70, Section 3.3]. Let $\vec{\mu}^*$ denote the optimal dual variable. We define $R \triangleq \|\vec{\mu}^*\|$ and $H \triangleq N \max_{i\in[N]} \left\{\sum_{\tilde{e}\in E_i} t_{\tilde{e}}^{ub} + T_i\right\}$. And we use $\vec{\mu}[i]$ to denote the dual variables in the $i$-th iteration. It is easy to verify that,

$$R \ge \left\|\vec{\mu}[0] - \vec{\mu}^*\right\| = \|\vec{\mu}^*\| = R, \tag{40}$$

and

$$H \ge \left\|\mathcal{D}'(\vec{\mu})\right\|, \tag{41}$$

where $\mathcal{D}'(\vec{\mu})$ is the sub-gradient of $\mathcal{D}(\vec{\mu})$ used in Alg. 1. And let $\phi$ be the constant step size we choose when we apply the sub-gradient based algorithm. We can derive that,

$$\begin{aligned}&\left\|\vec{\mu}[i+1] - \vec{\mu}^*\right\|^2 = \left\|\vec{\mu}[i] + \phi\mathcal{D}'(\vec{\mu}[i]) - \vec{\mu}^*\right\|^2 \\ &= \left\|\vec{\mu}[i] - \vec{\mu}^*\right\|^2 + 2\phi\mathcal{D}'(\vec{\mu}[i])^T\left(\vec{\mu}[i] - \vec{\mu}^*\right) + \phi^2\left\|\mathcal{D}'(\vec{\mu}[i])\right\|^2 \\ &\le \left\|\vec{\mu}[i] - \vec{\mu}^*\right\|^2 - 2\phi\left(\mathcal{D}^* - \mathcal{D}_i\right) + \phi^2\left\|\mathcal{D}'(\vec{\mu}[i])\right\|^2.\end{aligned} \tag{42}$$

The last inequality follows the definition of subgradient, which gives $\mathcal{D}^* - \mathcal{D}_i \le \mathcal{D}'(\vec{\mu}[i])^T\left(\vec{\mu}^* - \vec{\mu}[i]\right)$.

By applying (42) recursively, we get

$$\begin{aligned}\left\|\vec{\mu}[k+1] - \vec{\mu}^*\right\|^2 \le \left\|\vec{\mu}[1] - \vec{\mu}^*\right\|^2 - 2\phi\sum_{i=1}^{k}\left(\mathcal{D}^* - \mathcal{D}_i\right) \\ + \phi^2\sum_{i=1}^{k}\left\|\mathcal{D}'(\vec{\mu}[i])\right\|^2.\end{aligned} \tag{43}$$

By using $\left\|\vec{\mu}[i+1] - \vec{\mu}^*\right\|^2 \ge 0$, $\left\|\vec{\mu}[1] - \vec{\mu}^*\right\|^2 \le R^2$ and $H \ge \left\|\mathcal{D}'(\vec{\mu}[i])\right\|$, we have,

$$2\phi\sum_{i=1}^{k}\left(\mathcal{D}^* - \mathcal{D}_i\right) \le R^2 + \phi^2 k H^2. \tag{44}$$

Since $\mathcal{D}^* - \mathcal{D}_i \ge \mathcal{D}^* - \bar{\mathcal{D}}_k$, we have

$$\mathcal{D}^* - \bar{\mathcal{D}}_k \le \frac{R^2 + \phi^2 k H^2}{2\alpha k}. \tag{45}$$

We minimize the bound by setting $\phi = \frac{R}{H\sqrt{k}}$ and get

$$\mathcal{D}^* - \bar{\mathcal{D}}_k \le \frac{RH}{\sqrt{k}}. \tag{46}$$

However, the optimal dual variable $\vec{\mu}^*$ is unknown before we actually run the sub-gradient based algorithm. So we need to give an upper bound $R$ that is agnostic about the value of $\vec{\mu}^*$. Assume there exists a finite optimal solution $\vec{\mu}^*$ for the dual problem,

$$\mathcal{D}\left(\vec{\mu}^*\right) = \max_{\vec{\mu}} \mathcal{D}\left(\vec{\mu}\right) \ge \mathcal{D}\left(0\right). \tag{47}$$

Without loss of generality, we assume $\vec{x}^a$, $\vec{t}^a$ be a solution with fastest speed such that

$$\delta_i\left(\vec{x}^a, \vec{t}^a\right) = -T_i, \quad \forall i \in [N] \text{ that satisfies } \mu_i^* \ge 0, \tag{48}$$

$$\delta_i\left(\vec{x}^a, \vec{t}^a\right) > T_i, \quad \forall i \in [N] \text{ that satisfies } \mu_i^* < 0, \tag{49}$$

which means we do not spend time on edges in $E_i$ if $\mu_i^* \ge 0$ and we spend more than two times $T_i$ time on edges in $E_i$ if $\mu_i^* < 0$. Since

$$\mathcal{D}\left(\vec{\mu}^*\right) = \min_{\vec{x}\in\mathcal{X}} \min_{\vec{t}\in\mathcal{T}} \mathcal{L}\left(\vec{x}, \vec{t}, \vec{\mu}^*\right), \tag{50}$$

then we have

$$\mathcal{L}\left(\vec{x}^a, \vec{t}^a, \vec{\mu}^*\right) \ge \mathcal{D}\left(\vec{\mu}^*\right) \ge \mathcal{D}\left(0\right). \tag{51}$$

Let $\vec{x}^0, \vec{t}^0$ denote the optimal solution for $\min_{\vec{x}\in\mathcal{X}} \min_{\vec{t}\in\mathcal{T}} \mathcal{L}\left(\vec{x}, \vec{t}, \vec{0}\right)$. According to the definition of $\mathcal{L}\left(\vec{x}^a, \vec{t}^a, \vec{\lambda}^*\right)$, we have

$$C\left(\vec{x}^a, \vec{t}^a\right) + \sum_{i=1}^{N} \mu_i^*\left(\delta_i\left(\vec{x}^a, \vec{t}^a\right) - T_i\right) \ge C\left(\vec{x}^0, \vec{t}^0\right), \tag{52}$$

where

$$C\left(\vec{x}, \vec{t}\right) = \sum_{\tilde{e}\in E} c_{\tilde{e}}(t_{\tilde{e}}) x_{\tilde{e}}. \tag{53}$$

Since

$$\mu_i^*\left(\delta_i\left(\vec{x}^a, \vec{t}^a\right) - T_i\right) \le 0, \tag{54}$$

we can obtain a bound for $\mu_i^*$, which is

$$\mu_i^* \le \frac{C\left(\vec{x}^0, \vec{t}^0\right) - C\left(\vec{x}^a, \vec{t}^a\right)}{\delta_i\left(\vec{x}^a, \vec{t}^a\right)} \text{ if } \mu_i^* \ge 0; \tag{55}$$

$$\mu_i^* \ge \frac{C\left(\vec{x}^0, \vec{t}^0\right) - C\left(\vec{x}^a, \vec{t}^a\right)}{\delta_i\left(\vec{x}^a, \vec{t}^a\right)} \text{ if } \mu_i^* < 0. \tag{56}$$

Therefore, we have,

$$\begin{aligned}|\mu_i^*| &\le \frac{\left|C\left(\vec{x}^0, \vec{t}^0\right) - C\left(\vec{x}^a, \vec{t}^a\right)\right|}{\left|\delta_i\left(\vec{x}^a, \vec{t}^a\right)\right|} \\ &\le \frac{2}{T_i}\sum_{\tilde{e}\in\tilde{E}} \max_{t_{\tilde{e}}^{lb} \le t_{\tilde{e}} \le t_{\tilde{e}}^{ub}} c_{\tilde{e}}(t_{\tilde{e}}).\end{aligned} \tag{57}$$

Now we can give an upper bound of $\left\|\vec{\mu}[0] - \vec{\mu}^*\right\|$ that is agnostic about the value of $\vec{\mu}^*$

$$R = 2\sum_{i\in[N]} \frac{1}{T_i}\sum_{\tilde{e}\in\tilde{E}} \max_{t_{\tilde{e}}^{lb} \le t_{\tilde{e}} \le t_{\tilde{e}}^{ub}} c_{\tilde{e}}(t_{\tilde{e}}). \tag{58}$$

□

## APPENDIX C
## PROOF OF THM. 2

*Proof.* Let $\mathsf{sol}(\vec{\mu})$ denote the solution to $\mathsf{PSPV}$ and corresponds to dual variables $\vec{\mu}$, i.e., the solution with a path of $p^*(\vec{\mu})$ and a travel time profile of $t^*_{\tilde{e}}(\mu_i)$ assigned to each edge $\tilde{e} \in E_i$, for all $i \in [N]$.

Suppose $\vec{\mu}^*$ are dual variables that satisfy conditions (28). Satisfied conditions (28) imply following:

$$\sum_{\tilde{e}\in p^*(\vec{\mu}^*):\ \tilde{e}\in E_i} t^*_{\tilde{e}}(\mu_i^*) \;=\; \delta(\mu_i^*, \vec{\mu}^*) \;=\; T_i,\ \ \forall i \in [N], \tag{59}$$

i.e., the time-sensitive constraints (17b) of the formulation (17) are satisfied. Hence, $\mathsf{sol}(\vec{\mu}^*)$ is feasible to $\mathsf{PSPV}$ under phased speed ranges.

To prove the optimality of $\mathsf{sol}(\vec{\mu}^*)$, we first look at the value of the objective function (17a) of $\mathsf{sol}(\vec{\mu}^*)$:

$$\mathsf{Obj}(\mathsf{sol}(\vec{\mu}^*)) = \sum_{\tilde{e}\in p^*(\vec{\mu}^*)} c_{\tilde{e}}(t_{\tilde{e}}) = \sum_{i\in[N]} \sum_{\substack{\tilde{e}\in p^*(\vec{\mu}^*):\\ \tilde{e}\in E_i}} c_{\tilde{e}}(t^*_{\tilde{e}}(\mu_i^*)). \tag{60}$$

Next, we look at the value of the dual function corresponding to $\mathsf{sol}(\vec{\mu}^*)$:

$$\begin{aligned}
&\mathsf{Dual}(\mathsf{sol}(\vec{\mu}^*)) = \mathcal{D}(\vec{\mu}^*) \\
&= -\sum_{i\in[N]} \mu_i^* T_i + \sum_{\tilde{e}\in p^*(\vec{\mu}^*)} w_{\tilde{e}}(\vec{\mu}^*) \\
&= -\sum_{i\in[N]} \mu_i^* T_i + \sum_{i\in[N]} \sum_{\substack{\tilde{e}\in p^*(\vec{\mu}^*):\\ \tilde{e}\in E_i}} \left[c_{\tilde{e}}(t^*_{\tilde{e}}(\mu_i^*)) + \mu_i^* t^*_{\tilde{e}}(\mu_i^*)\right] \\
&= \sum_{i\in[N]} \sum_{\substack{\tilde{e}\in p^*(\vec{\mu}^*):\\ \tilde{e}\in E_i}} c_{\tilde{e}}(t^*_{\tilde{e}}(\mu_i^*)) \\
&\quad + \sum_{i\in[N]} \mu_i^* \cdot \left[-T_i + \sum_{\substack{\tilde{e}\in p^*(\vec{\mu}^*):\\ \tilde{e}\in E_i}} t^*_{\tilde{e}}(\mu_i^*)\right] \\
&\overset{(a)}{=} \sum_{i\in[N]} \sum_{\substack{\tilde{e}\in p^*(\vec{\mu}^*):\\ \tilde{e}\in E_i}} c_{\tilde{e}}(t^*_{\tilde{e}}(\mu_i^*)),
\end{aligned} \tag{61}$$

where the equality (a) comes from (59).

Now with equalities (60) and (61), we have:

$$\mathsf{Obj}(\mathsf{sol}(\vec{\mu}^*)) - \mathsf{Dual}(\mathsf{sol}(\vec{\mu}^*)) \;=\; 0, \tag{62}$$

i.e., the duality gap is zero, implying that $\mathsf{sol}(\vec{\mu}^*)$ must be an optimal solution to our $\mathsf{PSPV}$ problem under phased speed ranges. □

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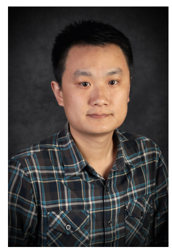

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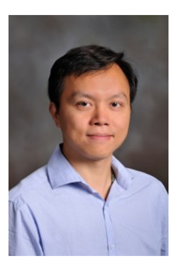

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