# Ride the Tide of Traffic Conditions: Opportunistic Driving Improves Energy Efficiency of Timely Truck Transportation 

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#### Abstract

We study the problem of minimizing fuel consumption of a heavyduty truck traveling across national highway subject to a deadline, under a practical setting that traversing a road segment is subject to variable speed ranges due to dynamic traffic conditions. The consideration of dynamic traffic conditions not only differentiates our work from existing ones but also allows us to leverage opportunistic driving to improve fuel efficiency. The idea is for the truck to strategically wait (e.g., at highway rest areas) for benign traffic conditions, to traverse subsequent road segments at favorable speeds for saving fuel. We observe that the traffic condition and thus the speed ranges are mostly stationary within certain durations of the day, and we term them as phases where each phase is defined as a time interval with fixed speed ranges. We formulate the fuel consumption minimization problem under phased speed ranges, considering path planning, speed planning, and opportunistic driving. We prove the problem is NP-hard, and develop a dual-subgradient heuristic for instances of the scale of national highway system. We characterize conditions under which the heuristic generates an optimal solution. We carry out simulations based on real-world traces over the US highway system. The results show that our scheme saves up to $26 \%$ fuel as compared to shortest-/fastest- path baselines, of which $11 \%$ is contributed by opportunistic driving. Meanwhile, opportunistic driving also reduces driving time by $13 \%$ as compared to only optimizing path planning and speed planning. As such, opportunistic driving offers a favorable design option to simultaneously reduce fuel consumption and hours of driving. Last but not least, our results highlight a perhaps surprising observation that dynamic traffic conditions can be exploited to achieve fuel savings even larger than those under stationary traffic conditions.


## CCS CONCEPTS

- Mathematics of computing $\rightarrow$ Paths and connectivity problems; • Applied computing $\rightarrow$ Transportation.

[^0]
## KEYWORDS

Energy-efficient transportation, timely transportation, opportunistic driving, dynamic traffic conditions, variable speed ranges

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## 1 INTRODUCTION

In 2017, the US trucking industry hauls $70 \%$ of all freight tonnage and collects $\$ 700.1$ billion in gross freight revenues [12]. The impressive revenue would rank 19 worldwide if measured against the GDPs of countries. Meanwhile, with only $4 \%$ of the total vehicle population, heavy-duty trucks consume $18 \%$ of energy in the whole transportation sector [26]. Furthermore, fuel consumption accounts for a significant fraction (26~34\%) of the truck operation cost [26]. These observations, together with that the global freight activity is predicted to increase by a factor of 2.4 by 2050 [13], make it critical to reduce fuel consumption for heavy-duty truck operation.

In this paper, we study an essential problem in truck operation, to minimize fuel consumption of a heavy-duty truck traveling across national highway subject to a deadline, under a practical setting that traversing a highway road segment is subject to variable speed ranges due to dynamic traffic conditions.

Transportation deadline: Freight delivery with time guarantee is common in truck operation; see examples and discussions in [4, 17]. As a more recent instance, mobile applications like Uber Freight provide many freight transportation tasks for truck operators, which are often associated with pickup/delivery time requirements.

Traffic condition: In practice, ranges of speed that a truck can travel on a highway depend on the traffic condition, especially on the highways close to urban areas. For example, during the rush hour, the traffic condition is harsh and one may only be able to drive at a speed much lower than the regulatory speed limit. In comparison, during the off-peak hours, the traffic conditions are benign and one can drive around the speed limit. The dynamic traffic conditions lead to variable speed ranges (VSR) of driving.

Path planning and speed planning are two well-recognized approaches to save fuel. Differences in distances and road conditions such as grade can lead to substantially different (e.g., $21 \%$ according
to [37]) fuel consumption when driving along different paths. Meanwhile, driving at an appropriate speed is also critical for saving fuel, considering that normally there is a most fuel-efficient speed for each vehicle. It is around 55 mph (mile per hour) on flat roads for many trucks [38], and the fuel economy will degrade if driving below or above this speed. As reported by [1, 21], every one mph increase in speed (above the most fuel-efficient speed) incurs about 0.14 mpg (mile per gallon) decrease in fuel economy. Here we highlight that road grade also plays an important role in speed planning because, with different grades, the fuel-rate-speed functions differ. As pointed out in [33], the diversity of grades brings potential for saving fuel without increasing trip time.

In addition to path planning and speed planning, the consideration of traffic conditions allows us to leverage opportunistic driving to improve fuel efficiency. The idea is for the truck to strategically wait for benign traffic conditions, to traverse subsequent road segments at favorable speeds for saving fuel. ${ }^{1}$ Specifically, in the US there are rest areas (see an example of Fig. 1a) with parking spaces next to highways, where drivers can rest, eat, or refuel without exiting onto secondary roads [40]. It can be more fuel economic for a truck driver to wait at certain rest areas for an appropriate duration, such that he/she can avoid the traffic rush hour and traverse subsequent road segments at favorable speeds for saving fuel. We note that in practice, usually truck drivers only need to deliver the loads to the destination before a given deadline; see e.g., the freight transportation requests on Uber Freight [8] and uShip [11]. Early arrival does not bring additional economic benefit. In such cases, drivers may choose to trade a longer trip time for fuel saving, as much as $26 \%$ comparing with conceivable fastest-path alternatives; see Sec. 6 for more details.

We give an illustrative example in Fig. 1b, and the optimal solution without (resp. with) opportunistic driving in Fig. 1d. Without opportunistic driving, the optimal solution is a path of $(B, C)$ with speed $\left\{r_{B}=50, r_{C}=40\right\}$, and the total fuel consumption is 3 . In comparison, with opportunistic driving, the optimal solution is a path of $(A, D)$ with speed $\left\{r_{A}=r_{D}=50\right\}$, and wait for one unit of time after passing $A$ before entering $D$, whose fuel consumption is 2. Note that opportunistic driving also allows us to reduce driving time from 2.25 to 2.0 in this example.

We also justify the temporal-spatial diversity of traffic condition using real-world data as follows. We utilize the fuel consumption model from [19, 20, 30] that is a function of driving speed. We select a road segment in the eastern US, and collect the road speed data using HERE map [7] (similar to those in Fig. 3). Fig. 2 shows that at 9 pm , the range of the driving speed corresponds to a less fuel-efficient part of the fuel-speed function. As a comparison, one hour later at 10 pm , the traffic condition improves and the speed range is more fuel-efficient, allowing the truck to travel at higher speeds and in the mean time reduce fuel consumption. As shown in Fig. 3, traffic condition also demonstrates spatial diversity, which can be readily exploited. By opportunistically scheduling the truck

[^1]
(a)

(b)

| Timeline | $[0,1)$ | $[1,2)$ | $[2,3)$ |
| :---: | :---: | :---: | :---: |
| Driving speed <br> range of A, B, C | $[30,50]$ | $[30,40]$ | $[30,50]$ |
| Driving speed <br> range of D | $[30,50]$ | $[30,35]$ | $[30,50]$ |

(c)

| Optimal solution <br> without | $s \xrightarrow{B, 50} a \xrightarrow{C, 40} d$ | 3.0 | 2.25 |
| :---: | :---: | :---: | :---: |
| opportunistic driving | Fuel cost | Driving time |  |
| Optimal solution with <br> opportunistic driving | $s, 50$ |  |  |
| opst $\rightarrow a \xrightarrow{D, 50} d$ | 2.0 | 2.0 |  |

(d)

Figure 1: (a) A highway rest area where trucks can strategically wait for benign traffic conditions. (b) A truck travels from $s$ to $d$ with a deadline $T=3$. The truck can wait at the end of road $A$ before entering $a$; it cannot wait after traversing road $B$. Each road has a length of 50 , and the corresponding truck fuel consumption rate function is $f(r)=$ $0.01 \times(r-50)^{2}+1$, where $r$ is the driving speed. (c) For roads $A, B$, and $C$ (resp. for road $D$ ), the speed range is [ 30,50 ] for $t \in[0,1)$ or $t \in[2,3)$ and is $[30,40]$ (resp. [30, 35]) otherwise. Speed ranges are different for roads $C$ and $D$ for $t \in[1,2)$. (d) The solutions show that opportunistic driving saves fuel and driving time simultaneously for this example.


Figure 2: The normalized fuel-speed function of a truck traversing a road segment. The most fuel-economic speed is about 32 mph with the truck full-loaded.
to traverse busy road segments (like road segments near a city) during the off-rush hour, or strategically park at rest areas or drive in free roads (like village roads) during rush hour, the truck can travel at energy-efficient speeds to save fuel.

Overall we observe that under dynamic traffic conditions, it is crucial to jointly consider path planning, speed planning, and

Table 1: Comparison of our work and existing studies for optimizing energy-efficient truck operations.

| Problem | Design Space |  |  | Problem Constraint |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Path Planning | Speed Planning | Opportunistic Driving | Deadline Constraint | Speed Range Constraint |
| RSP [24, 27, 31] | $\checkmark$ | $X$ | $X$ | $\checkmark$ | N/A |
| PASO [19, 20] | $\checkmark$ | $\checkmark$ | $X$ | $\checkmark$ | Static |
| TREK [30] | $\checkmark$ | $\checkmark$ | $X$ | $\checkmark$ | Static |
| Other, e.g., [25] | $X$ | $\checkmark$ | $X$ | $X$ | Static |
| Other, e.g., [18] | $\checkmark$ | $X$ | $X$ | $X$ | N/A |
| Our work | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | Variable |



Figure 3: We randomly select three road segments in the eastern US highway system, and plot the traffic speed (averaged over 10 days based on the speed data from HERE map [7]). We observe that traffic speed demonstrates certain "phase" properties. For example, all the traffic speeds are high during off-peak hours, e.g., from 3 am to 6 am , and are relatively low during peak hours e.g., from 9 am to 12 pm.
opportunistic driving to achieve maximum fuel saving. In this paper, we study a truck transportation problem of minimizing fuel consumption subject to a deadline constraint under this setting.

Existing studies. Energy efficient trucking has long been an active research area with efforts on various design options including path planning, speed planning, autonomous driving, and platooning, etc. To our best knowledge, this is the first to consider the design space of opportunistic driving by exploiting dynamic traffic conditions in addition to two important design spaces of path planning and speed planning. Tab. 1 compares our work with related studies.

PAth selection and Speed Optimization (PASO). PASO [19, 20] requires to figure out a path from a source to a destination, to minimize fuel consumption subject to a deadline constraint. It generalizes the problem of Restricted Shortest Path (RSP) [24, 27, 31] by additionally considering speed planning. The road speed planning is subject to a static speed range. This static model is less practical, because real-world speed range can change drastically with the dynamic traffic condition. Besides, there is no consideration of opportunistic driving in PASO.

Timely tRansportation for Energy-efficient trucKing (TREK). TREK [30] is an extension of PASO [19, 20]. Given many source-destination pairs, it requires to find a source-to-destination path for each pair, to minimize total fuel consumption subject to time window constraints of individual pairs. Under our system model where there is only one source-destination pair, TREK is equivalent to PASO.

Other related studies. Hellstrom et al. [25] develop an assistance system to use the predicted grade information to optimize driving
speed to save fuel, assuming a fixed path and hence no path planning is involved. Boriboonsomsin et al. [18] present an eco-routing navigation system that determines the most fuel-economic path, assuming fixed road driving speeds and hence no speed planning is involved. Moreover, existing studies [18,25] assume static road speed ranges, and do not consider opportunistic driving.

In addition to path planning and speed planning, there exist studies exploring other potentials, e.g., autonomous driving [23, 35] and vehicle platooning [14-16], for saving fuel. Here we remark that the design options explored by them $[14-16,23,35]$ for fuel consumption reduction differ from our options including path planning, speed planning, and opportunistic driving. Note that our solutions can serve as a critical building block for them, by providing an energy efficient path and speed profile for individual long-haul timely truck transportation.

In this paper we study the problem of minimizing fuel consumption for a truck to travel from a source to a destination under a deadline constraint. We simultaneously optimize path planning, speed planning, and opportunistic driving under VSR which comes from dynamic traffic conditions. We remark that our solution allows fuel saving under the condition that the truck does not miss deadline, where the deadline is an input to our problem. Hence, the driver can set the deadline according to their own considerations. Our specific contributions are as follows.
$\triangleright$ We observe that the traffic conditions and hence the speed ranges are mostly stationary within certain parts of the day, and we term them as phases where each phase is a time interval with fixed speed ranges. The concept of phase is consistent with the classical three-phase traffic theory [28], and we justify it by extensive simulations using real-world data. The concept of phase captures the dynamic characteristics of real-world traffic conditions, and in the mean time making our problem tractable. We formulate our problem under variable but phased speed ranges.
$\triangleright$ We prove that our problem is NP-hard. We exploit the structure of the dual of our phase-based problem formulation to design an efficient heuristic. Our heuristic can obtain high-quality solutions quickly for instances of the large-scale of national highway networks. We further derive sufficient conditions under which our heuristic generates an optimal solution.
$\triangleright$ We introduce the new design space opportunistic driving. In the existing studies of energy-efficient truck operations [19, 20, 30], saving energy is at the expense of increasing driving time and there seems to be a trade-off of energy efficiency and time efficiency. We show that driving time and fuel consumption can be reduced simultaneously by exploiting this new design space.
$\triangleright$ We carry out extensive simulations using real-world traces over the US national highway. Our solutions can save up to $26 \%$ fuel compared to fastest-/shortest- path baselines, among which $11 \%$ is contributed by opportunistic driving. Meanwhile, opportunistic driving also reduces driving time by $13 \%$ compared to only optimizing path and speed. As such, opportunistic driving offers a favorable design option for truck drivers to simultaneously reduce the fuel consumption and hours of driving. Simulation results also show that our solution, with a heuristic re-planning technique, is robust to the uncertainty in the traffic conditions and thus the speed ranges.

## 2 SYSTEM MODEL AND PROBLEM FORMULATION

### 2.1 System Model

We model a national highway network as a directed graph $G \triangleq$ $(V, E)$, where $E=E_{S} \cup E_{r}$. An edge $e \in E_{S}$ represents an actual road segment. An edge $e \in E_{r}$ is a virtual edge representing a rest area. Thus waiting at the rest area is modeled as traveling along a virtual edge. A node $v \in V$ represents a connecting point. For an edge $e=(u, v) \in E$, we denote head $(e)$ as its head, i.e., the node $v$, and denote tail $(e)$ as its tail, i.e., the node $u$.

Note that an actual road may correspond to multiple edges in our model. For example, for a 3-mile road with a rest area one mile away from the starting point of the road, we represent it by three edges in a sequence $\left\{e_{1}, e_{2}, e_{3}\right\}: e_{1} \in E_{s}$ corresponds to the first 1-mile segment of this road, $e_{2} \in E_{r}$ corresponds to the rest area located next to $e_{1}$, and $e_{3} \in E_{s}$ corresponds to the last 2-mile road segment. Since rest areas in practice usually locate next to the highway, we assume that the out-degree and in-degree are both equal to one for nodes head $(e)$ and tail $(e)$, for all virtual edges $e \in E_{r}$.

For each $e \in E_{s}$, we denote $D_{e}$ as its length, and denote $R_{e}^{l b}(t)$ (resp. $\left.R_{e}^{u b}(t)\right)$ as its minimum (resp. maximum) speed limit that depends on the time $t$ when the truck enters $e$. We remark that VSR, i.e., the speed ranges $\left[R_{e}^{l b}(t), R_{e}^{u b}(t)\right]$, differentiates our work from existing studies, e.g., $[19,20,30]$. It also allows us to introduce a new design space of opportunistic driving, i.e., in our system model we explicitly consider a rest edge set $E_{r}$ where the truck can strategically rest. Existing studies only consider stationary traffic conditions where $R_{e}^{l b}(t)=R_{e}^{l b}, R_{e}^{u b}(t)=R_{e}^{u b}$ for each $e \in E=E_{s}$, i.e., speed ranges are constants and time-invariant. While this is the case for highways in remote areas with little population and traffic, it does not model the dynamic traffic conditions that are common in urban or metropolitan areas; see an example in Fig. 3. In contrast, we capture the dynamics of traffic conditions by incorporating both space variance and time variance of speed range into our model.

In this paper we consider a fuel-efficient truck transportation problem under VSR. Truck fuel consumption is affected by many factors, e.g., driving speed and road grade. Like studies [19, 20, 30], we assume all the environment-specific factors such as road grade are fixed for each edge $e \in E$. Thus given an edge and the weight of the truck, the truck fuel consumption rate can be modeled as a convex function of driving speed. By the convexity, it is sufficient to maintain constant speed when driving along a road segment (see [19, Lem. 1]). Meanwhile, similar to [19, 20, 30], we neglect the time and fuel consumption of the acceleration/deceleration when a
truck changes its speed across adjacent edges, as this stage usually is only a few hundred meters long, and the corresponding time and fuel consumption are negligible compared to those of traveling along the road segment which is usually several miles long.

Overall, we denote $f_{e}\left(r_{e}\right)$ as the fuel consumption rate for the truck to pass an edge $e \in E$, at a constant speed of $r_{e}$. We assume that (i) $f_{e}\left(r_{e}\right)=0, \forall r_{e} \geq 0$ for any $e \in E_{r}$, and (ii) $f_{e}\left(r_{e}\right)$ is strictly convex in $r_{e}$ over a properly truncated speed range for any $e \in E_{S}$ (see the discussions in [19, 20, 30]). With the fuel-rate function $f_{e}\left(r_{e}\right)$, we can define the fuel consumption function $c_{e}\left(t_{e}\right)$ :

$$
c_{e}\left(t_{e}\right)= \begin{cases}t_{e} \cdot f_{e}\left(D_{e} / t_{e}\right), & \text { if } e \in E_{S} \\ 0, & \text { if } e \in E_{r}\end{cases}
$$

which is the fuel consumption for the truck to pass an edge $e \in E$, with a travel time of $t_{e}$.

### 2.2 Problem Formulation

We consider the scenario where a truck travels from a source $s \in V$ to a destination $d \in V$ over a national highway network $G$. Our objective is to minimize the total fuel consumption subject to VSR constraints and a deadline constraint. VSR constraints require that the truck driving speed at time $t$ on each edge $e \in E_{s}$ must be lower bounded by $R_{e}^{l b}(t)$ and upper bounded by $R_{e}^{u b}(t)$. The deadline constraint requires that the total travel time from $s$ to $d$, including the truck driving time on edges $e \in E_{S}$ and the truck waiting time on edges $e \in E_{r}$, must be no larger than a given deadline $T$.

The design space includes path planning, speed planning, and opportunistic driving. We introduce the following decision variables: variable $p$ defines a simple path from $s$ to $d$ over $G$, and variable $t_{e}$ defines the time for the truck to pass an edge $e \in E^{2}$. By vectoring variables as $\vec{t} \triangleq\left\{t_{e}: \forall e \in E\right\}$, our problem of optimizing Path planning, Speed Planning, and opportunistic driving under VSR (PSPV) has the following formulation:

$$
\begin{align*}
\min _{p \in \mathcal{P}, \vec{t} \in \mathcal{T}_{p}} & \sum_{e \in p} c_{e}\left(t_{e}\right)  \tag{1a}\\
\text { s.t. } & \sum_{e \in p} t_{e} \leq T \tag{1b}
\end{align*}
$$

where $\mathcal{P}$ is the set of simple paths from $s$ to $d$ in $G$, and $\mathcal{T}_{p}$ defines the set of possible travel times of all edges on the path $p$, i.e.,

$$
\begin{align*}
\mathcal{T}_{p} \triangleq\left\{\vec{t}: R_{e}^{l b}\left(\mathcal{S}_{e}(p, \vec{t})\right) \leq D_{e} / t_{e} \leq R_{e}^{u b}\left(\mathcal{S}_{e}(p, \vec{t})\right)\right. \\
\left.\forall e \in E_{S}: e \in p ; 0 \leq t_{e} \leq T, \forall e \in E_{r}: e \in p\right\} \tag{2}
\end{align*}
$$

$$
\text { where } \mathcal{S}_{e}(p, \vec{t})= \begin{cases}0, & \text { pre }(e, p) \text { is empty } \\ \sum_{e^{\prime} \in \operatorname{pre}(e, p)} t_{e^{\prime}}, & \text { otherwise }\end{cases}
$$

Here pre $(e, p)$ is the set of edges that are on the path $p$ and are precedent to the edge $e$, i.e., if the ordered edges of path $p$ is $\left\langle e_{1}, e_{2}, \ldots, e_{|p|}\right\rangle$, then we have:

$$
\operatorname{pre}\left(e_{k}, p\right)=\left\{e_{j}: \forall j=1,2, \ldots, k-1\right\}, \forall k=1,2, \ldots,|p| .
$$

With pre $(e, p), \mathcal{S}_{e}(p, \vec{t})$ is the starting time for the truck to pass the edge $e$ following the path $p$ and the edge travel time defined by $\vec{t}$.

[^2]In the formulation (1), objective (1a) minimizes the fuel consumption and constraint (1b) restricts the total travel time, including the total driving time and the total waiting time, to be bounded above by an input deadline $T$. Note that $\vec{t} \in \mathcal{T}_{p}$ are VSR constraints, which require the truck driving speed on each road be no smaller than the minimum road speed limit (resp. no greater than the maximum road speed limit) at the time when the truck enters the road.

We remark again that PSPV aims at saving fuel without missing the deadline, where the deadline is set by the user (e.g., trucking company, truck driver) according to their own considerations. As discussed in Sec. 1, in many real-world cases, truck drivers only need to deliver the freight to the destination before a given deadline and there is little economic incentive for drivers to arrive early. In these cases, drivers may choose to trade a longer trip time for fuel saving.

Our PSPV is NP-hard, since its special case PASO under stationary traffic conditions is NP-hard [19]. Thus it is impossible to obtain an optimal solution for PSPV in polynomial time, unless $\mathrm{P}=\mathrm{NP}$.

Proposition 2.1. Problem PSPV is NP-hard.

## 3 PHASE-DEPENDENT TRAFFIC CONDITION

We now develop a phase-based modeling approach. This idea is consistent with both the classical two-phase (i.e., free flow and congested traffic) traffic theory and the recent Kerner's three-phase (free flow, synchronized flow, and wide moving jam) theory [28, 29], although our concept of phase is defined in a rather general manner. We make the following two observations on the real-world traffic as illustrated in Fig. 3:

- while the traffic condition of a road segment may vary in time, it remains stationary for certain intervals, each with a length of several hours;
- different road segments in the same time zone may have different traffic conditions, but the transitions between different traffic conditions are pretty synchronous.
In the following, we define phases in traffic conditions.
Definition 3.1 (Phase [28, 29]). A (traffic) phase is a time interval during which the traffic conditions are stationary with fixed speed ranges for all road segments.

As seen later in Sec. 6, for properly defined phases, intra-phase speed variances of real-world road segments are substantially smaller than inter-phase speed variances. This observation suggests that the phase-based approach is suitable for modeling dynamic traffic conditions.

We now formulate PSPV under the setting of phase-based speed ranges. Let $t_{0}$ be the time for the truck to leave the source $s$. Suppose the time interval $\left(t_{0}, t_{0}+T\right)$ can be divided into $N$ phases, i.e.,

$$
\begin{equation*}
\left(t_{0}, t_{0}+T\right)=\bigcup_{i \in\{1,2, \ldots, N\}}\left(t_{0}+\sum_{j=0}^{i-1} T_{j}, t_{0}+\sum_{j=0}^{i} T_{j}\right), \tag{3}
\end{equation*}
$$

where phase $i$ starts at time $t_{0}+\sum_{j=0}^{i-1} T_{j}$ and ends at time $t_{0}+\sum_{j=0}^{i} T_{j}$. It is clear that $T_{i}$ is the length of phase $i$ and $\sum_{i=1}^{N} T_{i}=T$; we set $T_{0}=0$ for computation convenience. For each $i=1,2, \ldots, N$, we denote the fixed minimum speed limit (resp. fixed maximum speed
limit) of a road $e \in E_{S}$ at the phase $i$ by $R_{e}^{l b, i}$ (resp. $R_{e}^{u b, i}$ ), i.e.,

$$
R_{e}^{l b, i}=R_{e}^{l b}\left(t_{0}+\sum_{j=0}^{i-1} T_{j}\right), R_{e}^{u b, i}=R_{e}^{u b}\left(t_{0}+\sum_{j=0}^{i-1} T_{j}\right)
$$

We denote the road segment sequence of a path $p$ by $\left\langle e_{1}, e_{2}, \ldots, e_{|p|}\right\rangle$, where $e_{k} \in E$ is an edge on $p$ for each $k=1,2, \ldots,|p|$ and $|p|$ denotes the number of road segments on $p$. We can map the edge $e_{k} \in p$ and $e_{k} \in E_{s}$ to a phase $i \in\{1,2, \ldots, N\}$, once given a $p \in \mathcal{P}$ and a $\vec{t} \in \mathcal{T}_{p}$ : for each $k=1,2, \ldots,|p|$,

$$
I_{p, \vec{t}}\left(e_{k}\right)=i, \text { if } e_{k} \in E_{S} \text { and } \sum_{j=1}^{i-1} T_{j} \leq \sum_{j=1}^{k-1} t_{e_{j}}<\sum_{j=1}^{i} T_{j}
$$

Given phased speed ranges, we simplify the feasible set $\mathcal{T}_{p}$ in (2) to $\mathcal{T}_{p}^{\text {phase }}$ as follows:

$$
\begin{aligned}
\mathcal{T}_{p}^{\text {phase }} \triangleq & \left\{\vec{t}: R_{e}^{l b, I_{p, \vec{t}}(e)} \leq D_{e} / t_{e} \leq R_{e}^{u b, I_{p, \bar{t}}(e)},\right. \\
& \left.\forall e \in E_{s}: e \in p ; 0 \leq t_{e} \leq T, \forall e \in E_{r}: e \in p\right\} .
\end{aligned}
$$

Now PSPV under phased speed ranges can be formulated as:

$$
\begin{align*}
\min _{p \in \mathcal{P}, \vec{t} \in \mathcal{T}_{p}^{\text {phase }}} & \sum_{e \in p} c_{e}\left(t_{e}\right),  \tag{4a}\\
\text { s.t. } & \sum_{e \in p} t_{e} \leq T . \tag{4b}
\end{align*}
$$

Note that PSPV under phased speed ranges is NP-hard as it still covers the NP-hard problem PASO as a special case.

Proposition 3.2. PSPV under phased speed ranges is NP-hard.

## 4 PHASE-EXPANDED NETWORK AND PHASE-BASED REFORMULATION

In order to solve PSPV under phased speed ranges (i.e., the problem in (4)), in this section we construct a phase-expanded network and use it to reformulate PSPV. Our expanded-network-based formulation can be solved efficiently using a dual-subgradient-based heuristic that is introduced later in Sec. 5.

We construct the phase-expanded network $\tilde{G}(\tilde{V}, \tilde{E})$ from the input network $G(V, E)$ as follows: we let $\tilde{V}=\left\{v_{i}: \forall v \in V, \forall i \in\right.$ $[N]\}$ where $[N] \triangleq\{1,2,3, \ldots, N\}$ and $N$ is the number of phases. We let $\tilde{E}=H \cup L \cup R$, where $H$, $L$, and $R$ are sets of edges defined below, assuming $V_{r}=\left\{\right.$ head $\left.(e): \forall e \in E_{r}\right\} \cup\{d\}$ :

$$
\begin{aligned}
& H \triangleq \cup_{i \in[N]} H_{i}, \text { where } H_{i}=\left\{\left(u_{i}, v_{i}\right): \forall(u, v) \in E\right\}, \\
& L \triangleq \cup_{i \in[N-1]} L_{i} \text {, where } L_{i}=\left\{\left(v_{i}, v_{i+1}\right): \forall v \in V \backslash V_{r}\right\}, \\
& R \triangleq \cup_{i \in[N-1]} R_{i}, \text { where } R_{i}=\left\{\left(v_{i}, v_{i+1}\right): \forall v \in V_{r}\right\} .
\end{aligned}
$$

Here $v_{i} \in \tilde{V}$ denotes the node $v \in V$ in phase $i,\left(u_{i}, v_{i}\right) \in H_{i} \subseteq H$ denotes the edge $(u, v) \in E$ in phase $i$, and $\left(v_{i}, v_{i+1}\right) \in L_{i} \subseteq L$ (resp. $\left(v_{i}, v_{i+1}\right) \in R_{i} \subseteq R$ ) represents that a node $v \in V \backslash V_{r}$ (resp. a node $v \in V_{r}$ ) leaves phase $i$ and enters phase $i+1$.

We denote the minimum travel time (resp. maximum travel time) of an edge $\tilde{e} \in \tilde{E}$ as $t_{\tilde{e}}^{l b}$ (resp. $t_{\tilde{e}}^{u b}$ ). According to the respective


Figure 4: An example of constructing a phase-expanded network $\tilde{G}$ from an input network $G$. For $G$, we assume that $(a, b)$ is a waiting edge in $E_{r}$, while all the other edges are road segments in $E_{s}$. Suppose the source is $s$, destination is $d$, and deadline is $\sum_{i=1}^{4} T_{i}$ where $T_{i}$ is the length of phase $i$. Consider one solution in $G$, where one drives from $s$ to $a$ with a time of $T_{1}$, then waits from $a$ to $b$ with a time of $T_{2}$, and finally drives from $b$ to $d$ with a time of $T_{3}$. It corresponds to a solution in $\tilde{G}$ which allocates a set of travel time $\left\{0, T_{1}, 0,0, T_{2}, T_{3}, 0, T_{4}\right\}$ to edges of a $s$ - $d$ path $\left\{s, s_{1}, a_{1}, a_{2}, b_{2}, b_{3}, d_{3}, d_{4}, d\right\}$.
definitions of $H, L$, and $R$, we should have:

$$
\begin{aligned}
& t_{\tilde{e}}^{l b}= \begin{cases}D_{e} / R_{e}^{u b, i}, & \text { if } \tilde{e}=\left(u_{i}, v_{i}\right) \in H \text { and } e=(u, v) \in E_{S} \\
0, & \text { if } \tilde{e}=\left(u_{i}, v_{i}\right) \in H \text { and } e=(u, v) \in E_{r} \\
0, & \text { if } \tilde{e} \in L \cup R,\end{cases} \\
& t_{\tilde{e}}^{u b}= \begin{cases}D_{e} / R_{e}^{l b, i}, & \text { if } \tilde{e}=\left(u_{i}, v_{i}\right) \in H \text { and } e=(u, v) \in E_{S} \\
0, & \text { if } \tilde{e}=\left(u_{i}, v_{i}\right) \in H \text { and } e=(u, v) \in E_{r} \\
0, & \text { if } \tilde{e} \in L \\
T_{i}, & \text { if } \tilde{e} \in R\end{cases}
\end{aligned}
$$

For any $v \in V_{r}$, because one is allowed to wait at $v$ (either wait on the waiting edge $e \in E_{r}$ where head $(e)=v$ or wait at the destination $d$ after arriving ahead of deadline), we set the maximum travel time from $v_{i}$ to $v_{i+1}$ to be $T_{i}$, i.e., one can wait at $v$ to leave phase $i$ and enter phase $i+1$; otherwise for any $u \in V \backslash V_{r}$, we set the maximum travel time from $u_{i}$ to $u_{i+1}$ to be 0 , i.e., one cannot leave phase $i$ and enter phase $i+1$ if the arrival time at $u$ does not belong to the phase $i+1$. Note that we set the maximum travel time of the waiting edge in each phase (i.e., $\tilde{e}=\left(u_{i}, v_{i}\right) \in H$ and $e=(u, v) \in$ $E_{r}$, for $\left.i \in[N]\right)$ to be 0 , as we do not consider truck waiting inside each phase. This is because in practice rest areas are mainly used by the driver to wait for the ending of a phase with unfavourable traffic condition and enter a new phase with benign traffic condition.

For each $\tilde{e} \in \tilde{E}$, it has the following fuel consumption function:

$$
c_{\tilde{e}}(t)= \begin{cases}c_{e}(t), & \text { if } \tilde{e}=\left(u_{i}, v_{i}\right) \in H \text { and } e=(u, v) \in E, \\ 0, & \text { if } \tilde{e} \in L \cup R .\end{cases}
$$

We denote $E_{i}=H_{i} \cup L_{i} \cup R_{i}$, assuming $L_{N}=R_{N}=\emptyset$. We add an edge $\left(s, s_{1}\right)$ to $E_{1}$, and add an edge $\left(d_{N}, d\right)$ to $E_{N}$. For $\left(s, s_{1}\right)$ (resp. $\left(d_{N}, d\right)$ ), we set its minimum travel time and fuel consumption both to be 0 , and set its maximum travel time to be $0\left(\mathrm{resp} . T_{N}\right)$. Fig. 4
illustrates our phase-expanded network, where a feasible solution to problem (4) can be mapped to the phase-expanded network.

In the following we formulate our problem PSPV under phased speed ranges using the phase-expanded network as follows:

$$
\begin{align*}
\min _{\vec{x} \in \mathcal{X}, \vec{t} \in \mathcal{T}} & \sum_{\tilde{e} \in \tilde{E}} x_{\tilde{e}} \cdot c_{\tilde{e}}\left(t_{\tilde{e}}\right)  \tag{5a}\\
\text { s.t. } & \sum_{\tilde{e} \in E_{i}} x_{\tilde{e}} \cdot t_{\tilde{e}}=T_{i}, \forall i \in[N] \tag{5b}
\end{align*}
$$

where $t_{\tilde{e}}$ defines driving time (resp. waiting time) on $\tilde{e}$ if $\tilde{e}$ represents a road segment (resp. a rest area or the rest edge going into the destination). Though with equality constraint, we allow the truck arrive ahead of deadline no matter with or without opportunistic driving because we add a rest edge going into destination. $\mathcal{X}$ defines the set of feasible paths from $s$ to $d$ in the phase-expanded network,

$$
\begin{aligned}
X \triangleq & \left\{\vec{x}: x_{\tilde{e}} \in\{1,0\}, \forall \tilde{e} \in \tilde{E},\right. \text { and } \\
& \left.\sum_{\tilde{e} \in \operatorname{Out}(\tilde{v})} x_{\tilde{e}}-\sum_{\tilde{e} \in \ln (\tilde{v})} x_{\tilde{e}}=\mathbf{1}_{\tilde{v}=s}-\mathbf{1}_{\tilde{v}=d}, \forall \tilde{v} \in \tilde{V}\right\}
\end{aligned}
$$

where $1_{\{\cdot\}}$ is the indicator function, and $\operatorname{Out}(\tilde{v})=\{(\tilde{v}, \tilde{u}): \forall(\tilde{v}, \tilde{u}) \in$ $\tilde{E}\}($ resp. $\ln (\tilde{v})=\{(\tilde{u}, \tilde{v}): \forall(\tilde{u}, \tilde{v}) \in \tilde{E}\})$ is the set of outgoing edges (resp. incoming edges) of node $\tilde{v} \in \tilde{V} . \mathcal{T} \triangleq\left\{\vec{t}: t_{\tilde{e}}^{l b} \leq t_{\tilde{e}} \leq t_{\tilde{e}}^{u b}, \forall \tilde{e} \in\right.$ $\tilde{E}\}$ defines possible travel times of traversing each edge $\tilde{e} \in \tilde{E}$.

In the formulation (5), the objective (5a) minimizes the total fuel consumption and the time-aware constraints (5b) restricts that the aggregate travel time (recall that travel time includes both driving time and waiting time). Besides, since our phase-expanded network includes edges $\left(d_{i}, d_{i+1}\right)$ with a travel time upper bound of $T_{i}$, for each $i=1,2, \ldots, N-1$, our formulation naturally considers solutions of arriving at the destination $d$ ahead of deadline) of traversed edges in a phase should be equal to the length of this phase. We remark that for (5b) we give equality constraints instead of inequality constraints, because we fairly assume that the phase does not change while driving on any edge $e \in E_{S}$ and hence can only change when switching from one edge to another or waiting on a virtual edge $e \in E_{r}$. Our assumption comes from a practical concern that the length of a phase is much larger than the travel time of a real-world road segment (several hours compared to several minutes.). Hence, it is practically fair to leverage equality constraints so that we can map a solution in $\tilde{G}$ to a solution in $G$.

## 5 A SUBGRADIENT-BASED HEURISTIC FOR PSPV UNDER PHASED SPEED RANGES

In this section we design an efficient heuristic for PSPV under phased speed ranges. We try to obtain a high-quality solution iteratively, using the feedback of dual-subgradient information.

### 5.1 The Dual Problem

An important observation of problem (5) is that we can figure out the value of its dual by solving a shortest path problem, once given specific dual variables. Specifically, we first relax the time-sensitive
constraints (5b), and get the following Lagrangian function.

$$
\begin{aligned}
\mathcal{L}(\vec{x}, \vec{t}, \vec{\mu}) & =\sum_{i \in[N]} \sum_{\tilde{e} \in E_{i}} x_{\tilde{e}} c_{\tilde{e}}\left(t_{\tilde{e}}\right)+\sum_{i \in[N]} \mu_{i}\left(\sum_{\tilde{e} \in E_{i}} x_{\tilde{e}} t_{\tilde{e}}-T_{i}\right) \\
& =\sum_{i \in[N]} \sum_{\tilde{e} \in E_{i}} x_{\tilde{e}} \cdot\left(c_{\tilde{e}}\left(t_{\tilde{e}}\right)+\mu_{i} t_{\tilde{e}}\right)-\sum_{i \in[N]} \mu_{i} T_{i}
\end{aligned}
$$

Then corresponding dual function will be

$$
\mathcal{D}(\vec{\mu}) \triangleq \min _{\vec{x} \in \mathcal{X}, \vec{t} \in \mathcal{T}} \mathcal{L}(\vec{x}, \vec{t}, \vec{\mu})
$$

The dual problem of our problem (5) is: $\max _{\vec{\mu}} \mathcal{D}(\vec{\mu})$.
We can figure out the value of $\mathcal{D}(\vec{\mu})$ as follows

$$
\begin{aligned}
& \mathcal{D}(\vec{\mu})=\min _{\vec{x} \in X, \vec{t} \in \mathcal{T}} \mathcal{L}(\vec{x}, \vec{t}, \vec{\mu}) \\
= & -\sum_{i \in[N]} \mu_{i} T_{i}+\min _{\vec{x} \in \mathcal{X}, \vec{t} \in \mathcal{T}} \sum_{i \in[N]} \sum_{\tilde{e} \in E_{i}} x_{\tilde{e}} \cdot\left(c_{\tilde{e}}\left(t_{\tilde{e}}\right)+\mu_{i} t_{\tilde{e}}\right) \\
= & -\sum_{i \in[N]} \mu_{i} T_{i}+\min _{\vec{x} \in \mathcal{X}} \sum_{i \in[N]} \sum_{\tilde{e} \in E_{i}} x_{\tilde{e}} \min _{t_{\tilde{e}}^{l b} \leq t_{\tilde{e}} \leq t_{\tilde{e}}^{u b}}\left(c_{\tilde{e}}\left(t_{\tilde{e}}\right)+\mu_{i} t_{\tilde{e}}\right) .
\end{aligned}
$$

For each edge $\tilde{e} \in E_{i}$, suppose $t_{\tilde{e}}^{*}\left(\mu_{i}\right)$ is a feasible travel time which $\operatorname{minimizes} c_{\tilde{e}}\left(t_{\tilde{e}}\right)+\mu_{i} t_{\tilde{e}}$ for all $t_{\tilde{e}}^{l b} \leq t_{\tilde{e}} \leq t_{\tilde{e}}^{u b}$. Then we have

$$
\mathcal{D}(\vec{\mu})=-\sum_{i \in[N]} \mu_{i} T_{i}+\min _{\vec{x} \in \mathcal{X}} \sum_{i \in[N]} \sum_{\tilde{e} \in E_{i}} x_{\tilde{e}} \cdot\left[c_{\tilde{e}}\left(t_{\tilde{e}}^{*}\left(\mu_{i}\right)\right)+\mu_{i} t_{\tilde{e}}^{*}\left(\mu_{i}\right)\right]
$$

Now suppose we give a cost of $w_{\tilde{e}}(\vec{\mu})=c_{\tilde{e}}\left(t_{\tilde{e}}^{*}\left(\mu_{i}\right)\right)+\mu_{i} t_{\tilde{e}}^{*}\left(\mu_{i}\right)$ to each edge $\tilde{e} \in E_{i}$, for all $i \in[N]$. We have

$$
\mathcal{D}(\vec{\mu})=-\sum_{i \in[N]} \mu_{i} T_{i}+\min _{\vec{x} \in \mathcal{X}} \sum_{\tilde{e} \in \tilde{E}} x_{\tilde{e}} \cdot w_{\tilde{e}}(\vec{\mu})
$$

Assume $p^{*}(\vec{\mu})$ is the shortest path from $s$ to $d$ in the phase-expanded network, from the perspective of $w_{\tilde{e}}(\vec{\mu})$. Finally we have

$$
\begin{equation*}
\mathcal{D}(\vec{\mu})=-\sum_{i \in[N]} \mu_{i} T_{i}+\sum_{\tilde{e} \in p^{*}(\vec{\mu})} w_{\tilde{e}}(\vec{\mu}) \tag{6}
\end{equation*}
$$

According to (6), given specific dual variables $\vec{\mu}$, we can solve a shortest path problem in the phase-expanded network to figure out the value of its dual function $\mathcal{D}(\vec{\mu})$.

### 5.2 A Dual-Subgradient-Based Heuristic

We propose to iteratively update dual variables $\vec{\mu}$ using its subgradient, to minimize the duality gap and hence obtain high-quality solutions. The subgradient of the dual function in terms of each $\mu_{i} \in \vec{\mu}$ is denoted as $\mathcal{D}_{i}^{\prime}(\vec{\mu})$ and shown below

$$
\mathcal{D}_{i}^{\prime}(\vec{\mu})=-T_{i}+\sum_{\tilde{e} \in p^{*}(\vec{\mu}) \text { and } \tilde{e} \in E_{i}} t_{\tilde{e}}^{*}\left(\mu_{i}\right)
$$

We define $\delta_{i}(\vec{\mu})$ as the aggregate travel time of edges $\tilde{e} \in E_{i}$ and $\tilde{e} \in p^{*}(\vec{\mu})$, i.e., we define

$$
\delta_{i}(\vec{\mu}) \triangleq \sum_{\tilde{e} \in p^{*}(\vec{\mu}) \text { and } \tilde{e} \in E_{i}} t_{\tilde{e}}^{*}\left(\mu_{i}\right)
$$

The following lemma gives a critical property of $\delta_{i}(\vec{\mu})$, which is used to design the dual-subgradient-based heuristic.

Lemma 5.1. $\delta_{i}(\vec{\mu})$ is non-increasing with $\mu_{i}$.
Proof. Similar to the proof of [20, Thm. 3], and is skipped.


Figure 5: Simulated US national highway network. (a) Rest areas in US national highway network (with red marks). [9] (b) Simulated US regions (with red numbers) [19].

Based on Lem. 5.1, we can apply the following strategy to iteratively update $\vec{\mu}$ to obtain high-quality solutions:

$$
\begin{equation*}
\dot{\mu}_{i}=\phi\left(\mu_{i}\right) \cdot \mathcal{D}_{i}^{\prime}(\vec{\mu})=\phi\left(\mu_{i}\right) \cdot\left[\delta_{i}(\vec{\mu})-T_{i}\right], \forall i \in[N] \tag{7}
\end{equation*}
$$

where $\phi\left(\mu_{i}\right)$ is a step size to update $\mu_{i}$, which is positive due to Lem. 5.1. We present the details of our heuristic in Algorithm 1.

```
Algorithm 1 A Heuristic for PSPV under Phased Speed Ranges
    procedure
        Set sol \(=\) NULL, ite \(=1\), and \(\mu_{i}=0, \forall i \in[N]\).
        while ite \(\leq\) ITE_LIMIT do
            Get the solution sol' corresponding to \(\vec{\mu}\), which is the path
            \(p^{*}(\vec{\mu})\) with a travel time of \(t_{\tilde{e}}^{*}\left(\mu_{i}\right)\) assigned to each edge
            \(\tilde{e} \in E_{i}\), for all \(i \in[N]\). And let ite \(=\) ite +1
            Figure out \(\dot{\mu}_{i}\) according to (7), for all \(i \in[N]\)
            Let \(\mu_{i}=\mu_{i}+\dot{\mu}_{i}\), for all \(i \in[N]\)
            if \(\dot{\mu}_{i}=0, \forall i \in[N]\) then
                    return sol \(=\) sol'
            if sol' is feasible and saves fuel compared to sol then
                sol \(=\) sol \(^{\prime}\)
            return sol
```

In the following, we further introduce a set of complementary-slackness-like conditions under which the solution returned by our heuristic is optimal to our PSPV under phased speed ranges.

Theorem 5.2. If dual variables $\vec{\mu}$ satisfy

$$
\begin{equation*}
\delta_{i}(\vec{\mu})-T_{i}=0, \forall i \in[N] \tag{8}
\end{equation*}
$$

then it is an optimal solution to our PSPV under phased speed ranges, by following the associated path $p^{*}(\vec{\mu})$ with a travel time of $t_{\tilde{e}}^{*}\left(\mu_{i}\right)$ assigned to each edge $\tilde{e} \in E_{i}$, for all $i \in[N]$.

Proof. Refer to our technical report [6].
Overall, we design a heuristic (Algorithm 1) to solve PSPV under phased speed ranges. In order to obtain high-quality solutions, our heuristic tries to solve the dual problem of PSPV with the duality gap minimized, by iteratively updating dual variables using the dual-subgradient information. We further derive conditions under which our heuristic outputs an optimal solution (Thm. 5.2).

## 6 PERFORMANCE EVALUATION

We use real-world traces to evaluate our heuristic. Our experiments are implemented using $\mathrm{C}++$ and python and run on a server cluster with 42 processors, each equipped with 20 GB memory on average. We represent a PSPV instance by a tuple of $\left(s, d, T, t_{0}\right)$, where $s$ is source, $d$ is destination, $T$ is deadline, and $t_{0}$ is truck departure time.

Transportation network and heavy-duty truck. We construct the US national highway network from the Clinched Highway Mapping Project [36], and focus on its eastern part with 38213 connecting points and 82781 directed road segments. As illustrated in Fig. 5b, we divide eastern US into 22 regions, where later the source and destination of the truck are nodes that are nearest to certain selected regions' center. Our simulated truck is a class-8 heavy-duty truck Kenworth T800, with 36 -ton full load [3].

Rest edges. We randomly select a fraction of road segments and assume there is a rest area with each of them. The rest area density $\rho$ is defined as the ratio of the number of rest areas to that of road segments. Noticing there are 1906 rest areas with truck parking slots in the highway network of eastern US [5], we estimate the real-world rest area density to be roughly $\frac{1906}{82781}=0.023$. We set the rest area density in our simulation to be in the same scale.

Variable speed ranges. To model road speed ranges which depend on the dynamic traffic condition, we collect real-time road speed data from HERE map [7] for 10 days ( $08 / 18 / 2017$ - 08/27/2017). We divide one day into 8 phases evenly. Real-world traffic statistics show that the inter-phase speed variance is about 12 times the intra-phase speed variance in average. Thus the concept of phase captures the main characteristic of real-world traffic condition. For each phase, we use the average road traffic speed as the road maximum speed limit in this phase. And we set the minimum road speed limit to be the minimum of 15 mph and the average speed.

Road fuel consumption. We first obtain the grade of each road segment based on the elevations of nodes provided by the Elevation Point Query Service [10]. We then use the ADVISOR simulator [32] to collect fuel consumption rate data with the truck driving speed for different road grade. Finally we use MATLAB to fit the fuel consumption rate function of speed using a 3 -order polynomial given a specific road grade. The same model of fuel consumption rate function has been used in related studies [19, 20, 30].

### 6.1 Reduction on both Fuel Consumption and Driving Time by Opportunistic Driving

Everyday driving experience and existing results [19, 20] seem to imply that a trade-off between the time efficiency and energy efficiency in vehicle transportation exists. However, we observe that this trade-off is not the whole picture. As discussed in Sec. 2.1 and illustrated by Fig. 2, the fuel consumption of traversing a road segment is a convex function that first decreases and then increases with the driving speed. In the decreasing part, increasing speed benefits both energy saving and time saving, while in the increasing part, fuel saving is at the expense of increasing driving time. Our solution allows a truck to opportunistically traverse busy road segments at a favourable speed. Therefore, the truck can save both time and fuel of traversing such road segments instead of saving fuel at the expense of increasing driving time. In contrast, existing
state-of-the-art solution PASO $[19,20]$ assumes static speed ranges and saves fuel at the expense of increasing hours of driving.

A conceivable approach that generalizes PASO to our setting is as follows: we first consider a static-speed-range-setting where we set the minimum speed limit (resp. maximum speed limit) to be the average minimum speed limit (resp. average maximum speed limit) in all phases. Thus we get an instance of PASO which can be solved by the existing heuristic from [19]. We then round the solution in that if the driving speed violates the variable road speed range, we reset it to be the maximum variable speed limit.


Figure 6: We set $s=14, d=22$, and $t_{0}=5 \mathrm{AM}$. (a) The conceivable approach can miss the deadline, while our solution always satisfies the deadline constraint. (b) Our solution achieves about $10 \%$ fuel saving than conceivable approach while using less driving time. This set of results highlight that opportunistic driving not only saves fuel but also reduces driving time as compared to existing solutions without opportunistic driving optimization.

Fig. 6 gives simulation results which suggest that our solution significantly saves both fuel and driving time as compared to existing solutions. This is because existing solutions assume static speed ranges and hence do not explore opportunistic driving to save fuel.

### 6.2 Performance Evaluation of Our Heuristic using Extensive Simulations

We conduct extensive simulations to evaluate our heuristic, compared to several baselines. For the shortest/fastest-path baseline, we drive as fast as possible along the shortest/fastest ${ }^{3}$ path. We choose the density of rest areas between 0.0 and 0.1 . We sample 25 different $(s, d)$ pairs which evenly cover the eastern US. Given a $(s, d)$ pair, we randomly select a departure time $t_{0}$ from 0 AM to 11 PM with a step of 1 hour, and randomly select a deadline $T$ from $1.3 \cdot T_{\min }$ to $2.0 \cdot T_{\min }$ with a step of $0.1 \cdot T_{\min }\left(T_{\min }\right.$ is the minimum travelling time from source to destination under average static speed ranges). Some instances are inherently infeasible due to the bad traffic condition. We finally simulate a total of 840 feasible instances and present the simulation results in average in Tab. 2. We observe that our heuristic saves $26 \%$ fuel compared to the fastest-/shortestpath baselines, where $11 \%$ is contributed by opportunistic driving; and our solution with opportunistic driving saves up to $13 \%$ driving time as compared to solutions without opportunistic driving.

We highlight again that optimizing opportunistic driving's benefit for truck operators is two-fold. First, it allows one to efficiently save fuel. Second, it helps to reduce driving time. We also remark

[^3]Table 2: Average performance of different algorithms (the average deadline is 19.8 hours).

|  | Fastest <br> baseline | Shortest <br> baseline | Our heuristic <br> (without OD) | Our heuristic <br> (with OD) |
| :---: | :---: | :---: | :---: | :---: |
| Fuel (gallon) | 93.5 | 93.5 | 79.7 | 69.4 |
| Time (hour) | 11.7 | 11.8 | 18.7 | 16.2 |



Figure 7: (a) Fuel consumption of our heuristic for the instance of $s=16, d=17, T=11$, with different departure times $t_{0}$. (b) Fuel consumption of our heuristic for the instance of $s=21, d=19, t_{0}=2 \mathbf{P M}$, with different deadlines $T$.
that another unexpected potential benefit is that traffic congestion is relieved by letting trucks wait in a busy phase and drive in a benign phase. Therefore, our solution is a win-win-win situation.

### 6.3 Impact of Truck Departure Time and Deadline on Fuel Consumption

Under the variable-speed-range-setting, it is intuitively that one can reduce fuel consumption if he/she leaves the source at a time without traffic congestion. Now we show some observations of the impacts of the departure time $t_{0}$ on the fuel consumption.

Fig. 7a gives the fuel consumption of different $t_{0}$, with $s=16$, $d=17$, and $T=11$. We observe that there is a morning peak and an evening valley, which coincides with our experience that driving at evening usually consumes less fuel, when the traffic is more free. Besides, we verify again that opportunistic driving can save fuel, where the fuel consumption of our heuristic with rest areas ( $\rho=0.05$ ) is less than that without rest areas $(\rho=0.0)$. We also observe that perhaps interestingly, the morning peak and evening valley phenomena are relieved when we allow opportunistic driving. Again the reason is that we are able to opportunistically traversing those congested roads off the rush hour. Next we show the impact of the input deadline. Fig. 7b gives the fuel consumptions of our heuristic of different deadlines $T$, with $s=21, d=19$, and $t_{0}=$ 2PM. We observe that fuel consumption decreases a lot as deadline increases for the results without opportunistic driving. However, the fuel consumption with opportunistic driving does not decrease in this special case. Overall, the above observations again verify that opportunistic driving significantly saves more fuel than only optimizing path and speed. The performance gain does not come from simply increasing driving time but from opportunistic driving.

### 6.4 Dynamic vs. Static Traffic Conditions

In this subsection, we highlight a perhaps surprising observation that dynamic traffic conditions (and hence variable speed ranges)


Figure 8: Ratio comparing the fuel consumption of the setting of variable speed ranges to that of the setting of static speed ranges, with respect to the input deadline.
can offer more fuel saving potential than static traffic conditions. Opportunistic driving can allow us to capitalize such a potential.

We set the static speed range of each road as the average of the variable speed ranges. We run our heuristic to obtain the fuel consumption under the setting of variable speed ranges, while we use the existing heuristic from $[19,20]$ to obtain the fuel consumption under the setting of static speed ranges. Fig. 8 presents the fuel-consumption-ratio results averaged over 300 feasible instances. We observe that for small delay factor $T / T_{\min }$, static speed ranges can save more fuel, due to that the transportation time urgency admits little room for opportunistically driving. However, as the delay factor increases, opportunistic driving can allow us to ride the tide of variable speed ranges to achieve a larger fuel saving than that under the static speed range. The difference is as much as $3 \%$ in this evaluation. Further, as the delay factor increases, we also observe that without opportunistic driving, the fuel-consumption-ratio converges to a value around one; but with opportunistic driving, the ratio is strictly decreasing. It verifies again that opportunistic driving is critical for saving fuel under variable speed ranges.

### 6.5 Discussions

Side-effect on traffic condition. Intuitively, the traffic condition may be affected if many trucks follow our solutions. In fact, such influences are minor, since the number of trucks only accounts for a small portion of the total vehicle population: in the US in 2015 there are 263.6 million vehicles [39] of which 15.5 million are trucks [2].

Robustness to real-world speed perturbation. The phased speed range is just a prediction of the future speed range. Real speed range may be perturbed by random factors. However, such perturbation is in a much smaller time and strength scale than our phased speed range, thus cannot make significant effect. We verify it in our simulation. We use a simple "re-balance" heuristic that we round our assigned speed to real speed range and rest less or run faster if we are behind the time schedule. We find that only less than $1 \%$ of the instances miss the deadline, where the deadline violation ratio is just $0.6 \%$ on average. The increase on fuel consumption caused by real-world speed range perturbation is only $0.8 \%$.

Incorporating real-time traffic condition. Note that nowadays traffic condition can be forecasted more accurately in real time [34]. When implementing our solution in practice, we can tune both the phase length and traffic speed range based on the forecasting result at runtime, solve our problem using updated information,
and refine the obtained solution which optimizes path planning, speed planning, and opportunistic driving.

As discussed in Sec. 1, in the literature there are research efforts on various design options including path planning [18-20, 24, 27, 30,31 ], speed planning [19, 20, 25, 30], autonomous driving [23, 35], and platooning [14-16], etc. Our study optimizes path planning, speed planning, and opportunistic driving. It can serve as a critical building block for other studies, by providing a fuel-efficient path and speed profile follow which the truck can travel from a source to a destination over a national highway. Overall, our study can be of great interest to the truck operators by timely delivering freight and reducing fuel consumption.

## 7 CONCLUSION AND FUTURE WORK

We study the problem of minimizing fuel consumption of a heavyduty truck traveling across national highway subject to a deadline, where traversing a road is subject to variable speed ranges due to dynamic traffic conditions. The consideration of dynamic traffic condition differentiates our study from existing ones, and allows us to leverage on opportunistic driving to save fuel. We observe that real-world speed ranges are largely phase-dependent, where a phase is a time interval with static traffic conditions and hence fixed speed ranges. We prove that our problem under phased speed ranges is NP-hard, and give a phase-based formulation to it. By exploiting its dual problem, we develop an efficient dual-subgradient heuristic, which generates optimal solutions under derived conditions. We conduct extensive simulations using real-world traces over the US highway system, and observe that our heuristic can save up to $26 \%$ fuel compared to fastest-/shortest- path baselines, among which $11 \%$ is contributed by opportunistic driving. Meanwhile, opportunistic driving also reduces driving time by $13 \%$ as compared to only optimizing path planning and speed planning. As such, opportunistic driving offers a desirable design option to simultaneously reduce fuel consumption and hours of driving. Last but not least, our results highlight a perhaps surprising observation that dynamic traffic conditions can be exploited to achieve fuel savings even larger than those under stationary traffic conditions. It is an interesting future direction to study the influence of opportunistic driving on traffic conditions.

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[^1]:    ${ }^{1}$ At a high level, the idea is similar to the opportunistic scheduling widely used in commercial cellular wireless systems [22], in the sense that both aim at exploiting variation in the environment to boost system performance. In wireless communication, opportunistic scheduling exploits the variation in the communication channel gain to increase the throughput. In our truck transportation scenario, we propose to leverage on the dynamic traffic conditions to drive opportunistically to reduce fuel consumption.

[^2]:    ${ }^{2}$ If $e \in E_{S}, t_{e}$ is driving time; otherwise if $e \in E_{r}, t_{e}$ represents waiting time.

[^3]:    ${ }^{3}$ We derive the path under average static speed range.

